ANALYTICITY OF SETS ASSOCIATED TO LELONG NUMBERS AND THE EXTENSION OF MEROMORPHIC MAPS¹

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We announce the following two theorems and sketch the proofs of some special cases which are indicative of the ideas of the complete proofs.

THEOREM I. If u is a closed positive (k, k) current on an open subset of C^n , then for $c \ge 0$ the set of points where the Lelong number of u is $\ge c$ is an analytic set.

THEOREM II. Let X be a normal complex space, A be an analytic set of codimension ≥ 1 in X, and G be an open subset of X which intersects every branch of A of codimension 1. If M is a compact Kähler manifold, then every meromorphic map f (in the sense of Remmert) from $(X - A) \cup G$ to M can be extended to a meromorphic map from X to M.

Theorem I has been an outstanding conjecture. This author first heard the conjecture mentioned by Lelong in the 1972 Several Complex Variables Conference in Poitiers, France, and later learned that it was also mentioned in $\lceil 6 \rceil$ and by Harvey in the 1971 Berkeley Summer Conference on Partial Differential Equations. Some partial results in this direction have been obtained. This [12] proved that, when u is defined by integration over an analytic subset of pure codimension k, the Lelong number agrees with the multiplicity of the analytic set and hence the conjecture is true in this case. King [7] proved that, if the Lelong number of u is an integer at almost every point of its support with respect to the Hausdorff (2n - 2k)measure, then u is a linear combination of currents defined by integration over analytic subsets of pure codimension k, and, as a consequence, the conjecture is true in this case. Harvey and King [6] improved the result of [7] and showed that if, for every compact subset K of Supp u, there exists c > 0 such that the Lelong number of u at x is $\geq c$ for almost all $x \in K$ with respect to the Hausdorff (2n - 2k)-measure, then u is a linear combination of currents defined by integration over analytic subsets of pure codimension k, and, consequently, the conjecture is true in this case. Recently Skoda [11] proved that, if the domain D where u is defined is Stein, then for every $c \ge 0$ there exists an analytic subset X_c of D such that

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