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SELFCOMMUTATORS OF MULTICYCLIC HYPONORMAL OPERATORS ARE ALWAYS TRACE CLASS

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1. For A, B operators on the Hilbert space H, $[A, B] = AB - BA$. The selfcommutator of A is $[A^*, A]$. If E is a closed proper subset of the plane, $R(E)$ will be the rational functions analytic on E. The operator A is said to be n-multicyclic if there are n vectors $g_1, \ldots, g_n \in H$, called generating vectors, such that $\{r(A)g_i : r \in R(\mathrm{sp}(A)), 1 \leqq i \leqq n\}$ has span dense in H. This paper will outline a circle of ideas culminating in the following result.

MAIN THEOREM. *If A is an n-multicyclic hyponormal operator, then $[A^*, A]$ is in trace class, and* $\mathrm{tr}[A^*, A] \leqq (n/\pi)\omega(\mathrm{sp}(A))$, *where ω is planar Lebesgue measure.*

This result is especially interesting because of the scarcity of known conditions insuring that the selfcommutator lie in trace class. The above result is new even when A is subnormal and has a cyclic vector in the usual sense. The best previous result in this direction is due to T. Kato [**1**], and states that if $\mathrm{Re}(A)$ has finite spectral multiplicity n, then $[A^*, A]$ is in trace class. Kato provides a trace estimate which Putnam [**4**] is able to use to prove the above estimate, where n is an upper bound for the spectral multiplicity of $\mathrm{Re}(A)$.

The Kato-Putnam estimate and the main theorem above are independent. For example, using a result of J. W. Helton and R. Howe, unpublished as yet, which provides a lower bound for the spectral multiplicity of the real part of a hyponormal operator, one can see that the real part of the 1-multicyclic operator given by multiplication by z on R^2 of a Swiss cheese has infinite spectral multiplicity almost everywhere.

Throughout the following, a space and the orthogonal projection onto that space will be denoted by the same symbol. All spaces are Hilbert spaces.

2. The following lemma is central.

STRUCTURE LEMMA. *Let T and A be hyponormal operators on H and K*

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