BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 79, Number 5, September 1973

HOLOMORPHIC FOCK REPRESENTATIONS AND PARTIAL DIFFERENTIAL EQUATIONS ON COUNTABLY HILBERT SPACES

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Communicated by Philip Hartman, January 30, 1973

1. **Introduction.** Differential operators on the holomorphic Fock space $F(E)$ of entire functions of Hilbert-Schmidt type on a Hilbert space E of [**Be**], [**D1**] and [**R**] describe observables of quantum systems with an infinite number of degrees of freedom [**Be**]. These operators are in general unbounded on $F(E)$, leading to the introduction of weighted holomorphic Fock spaces and their projective and inductive limits, on which suitable differential operators are bounded [**D1**], [**D2**] and [**R**]. For certain weights, existence theorems also hold [**D1**]. However, the weighted spaces are not nuclear on infinite-dimensional Hilbert space domains, so kernel representations do not in general hold. In this note we introduce instead holomorphic Fock spaces on countably Hilbert spaces and on their duals (Theorems 2.2 and 2.3), with suitably defined Hilbert-Schmidt polynomial derivatives (Theorem 2.1), again obtaining boundedness and existence theorems (Theorems 3.1 and 3.2), as well as nuclearity under certain conditions (Proposition 2.1). Moreover, these function spaces provide representations of tempered distributions in infinite dimension in the sense of [**KMP**].

2. **Holomorphic mappings on countably Hilbert spaces.** Let E be a projective limit of Hilbert spaces E_r with injective and dense linear maps $E_s \to E_r$ for $r \leqq s$ (real or integer indices), and E' its strong dual. By $\boldsymbol{P}_H({}^nE_r)$ and similarly on E_r', we mean the spaces of n-homogeneous Hilbert-Schmidt polynomials as in [**D1**], [**D2**]. We recall that $\boldsymbol{P}_H({}^nE_r')$ and $\boldsymbol{P}_H({}^nE_r)$ are in duality for a bilinear form $\langle\ ,\ \rangle_n$ such that $\langle x^n, x'^n\rangle_n = \langle x, x'\rangle^n$. The Hilbert-Schmidt norm on $\boldsymbol{P}_H({}^nE_r')$ is denoted by $\|\ \ \|_{r,n}$ and on $\boldsymbol{P}_H({}^nE_r)$ by $\|\ \ \|_{r,n}'$. We define the spaces of n-homogeneous Hilbert-Schmidt polynomials on E' and E to be respectively the projective and inductive limits $\boldsymbol{P}_H({}^nE') = \lim \mathrm{inv}_r\, \boldsymbol{P}_H({}^nE_r')$ and $\boldsymbol{P}_H({}^nE) = \lim \mathrm{dir}_r\, \boldsymbol{P}_H({}^nE_r)$, for the canonical extension and restriction maps. By $\boldsymbol{F}(E_r')$ we mean the Hilbert space of entire functions f of Hilbert-Schmidt type on E_r' such that

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AMS (MOS) subject classifications (1970). Primary 35R15, 58B10, 32K05; Secondary 47B10, 46E10, 81A17.

Key words and phrases. Fock representations, infinite-dimensional holomorphy, infinite-dimensional distributions, infinite-dimensional partial differential equations.

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