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EXTREMAL POSITIVE SPLINES WITH APPLICATIONS TO INTERPOLATION AND APPROXIMATION BY GENERALIZED CONVEX FUNCTIONS

BY HERMANN G. BURCHARD[1]

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1. **Preliminaries.** Consider an nth order Chebyshevian, or disconjugate, differential operator $L = w_0^{-1}Dw_1^{-1} \cdots Dw_n^{-1}$, where $D = d/dt$ and $w_i \in C^n(\boldsymbol{R})$, $w_i(t) > 0$, $i = 0, 1, \ldots, n$, $t \in \boldsymbol{R}$ [4], [5]. The formal adjoint of L is the Chebyshevian operator $L^* = (-1)^n w_n^{-1}Dw_{n-1}^{-1} \cdots Dw_0^{-1}$. Let

$$F(s, t) = w_n(s)w_0(t)\int_t^s dt_1 w_1(t_1)\int_{t_1}^s dt_2 w_2(t_2) \cdots \int_{t_{n-2}}^s dt_{n-1}w_{n-1}(t_{n-1}).$$

A fundamental solution for L is given by $G(s, t) = F(s, t)$ for $s \geqq t$, $G(s, t) = 0$ for $s < t$. A fundamental solution for L^* is $G_*(s, t) = G(t, s)$. To avoid cumbersome formulations results are stated for $n \geqq 2$ (in which case G is continuous), unless indicated otherwise.

By $\mathfrak{M}(T)$ we mean the collection of Radon measures on the locally compact Hausdorff space T; by $\mathfrak{M}_0(T)$ and $\mathfrak{M}(T)^+$, the subfamilies of measures of compact support and of positive measures. For an open interval I let $\mathfrak{M}^n(I)$ be the set of real functions on I possessing an nth distribution derivative belonging to $\mathfrak{M}(I)$, $n = 1, 2, \ldots$. For $n \geqq 2$, if $u \in \mathfrak{M}^n(I)$ then $D^{n-2}u \in AC^{\mathrm{loc}}(I)$ and $D^{n-1}u \in BV^{\mathrm{loc}}(I)$. One shows that a measure $u \in \mathfrak{M}(I)$ belongs to $\mathfrak{M}^n(I)$ iff $Lu \in \mathfrak{M}(I)$ in the following weak sense: There is $\mu \in \mathfrak{M}(I)$ such that $\int L^*\phi(t)u(dt) = \int \phi(t)\mu(dt)$ for each $\phi \in C_0^n(I)$; if this is so one says $\mu = Lu$. Let $\mathfrak{M}_0^n(\boldsymbol{R})$ consist of the functions in $\mathfrak{M}^n(\boldsymbol{R})$ of compact support. For any interval I we say $u \in \mathfrak{M}_{00}^n(I)$ if $u \in \mathfrak{M}_0^n(\boldsymbol{R})$ and $\mathrm{supp}(u) \subset I$. Each $u \in \mathfrak{M}^n(I)$ has integral representations

$$u(s) = v(s) + \int_a^s F(s, t)Lu(dt), \quad \text{where } Lv = 0 \text{ and } a \in I,$$

and analogously

$$u(t) = v^*(t) + \int_a^t F(s, t)L^*u(ds), \quad \text{with } L^*v^* = 0.$$

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