

BOOK REVIEWS

Free rings and their relations, by P. M. Cohn. Academic Press, London & New York, 1971. xvi + 346 pp. \$22.00

Free associative algebras, i.e., polynomial rings in noncommuting variables, are not mentioned in most of the standard texts in ring theory; Cohn's book is the first comprehensive treatment of this subject. The book is up-to-date, very well written and essentially self-contained.

The lack of reference to free rings in previous works is understandable; these rings stand somewhat apart from the traditional branches of ring theory. The lack of finiteness conditions in free algebras (in fact they are automatically infinite dimensional) dissociates them from the classical theory of noncommutative algebras which were always assumed to be finite dimensional vector spaces over the base field. On the other hand, the noncommutativity of free rings separates them from the other main branch of ring theory: algebraic number fields and their generalizations.

Recently, a new way of approaching problems in algebraic geometry has provided a geometric insight into many notions of ring theory. But the commutativity restrictions cannot be easily circumvented. Noncommutative algebraic geometry is almost nonexistent, and before such a theory could be done one would have to develop a theory of algebraic equations in noncommuting indeterminates. At present, not enough is known about the structure of free rings, and we are far from being able to handle some of the most elementary problems that occur in this field. Most of the questions one asks are simple translations from the commutative case and from the theory of free groups. Unfortunately, the methods cannot be successfully adapted in most cases, and the translated "theorems" are not even true.

The systematic development of the subject has been done mainly by P. M. Cohn during the last decade. Many of the theorems in this book are taken directly from the author's papers and from G. M. Bergman's thesis (Harvard 1967), in which some outstanding open problems were solved and existing results were simplified and generalized. Though this book deals with a very specialized subject within ring theory, the author has been careful, at every stage, to establish the connection with the traditional branches by showing how many of the standard notions can be generalized.

The book does not contain a complete account of related topics (in fact, a second volume is mentioned). For example the construction of free products is not included. Aside from some minor misprints I have not found any major errors in the text.