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EUCLIDEAN SUBRINGS OF GLOBAL FIELDS[1]

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1. **Introduction.** The purpose of this note is to announce some results regarding the existence of euclidean subrings of global fields.

We first state the problem and give its history. Let F be a global field. So F is a finite extension of the rational numbers Q or F is a function field of one variable over a finite field k, where k is algebraically closed in F. Let S be a finite nonempty set of prime divisors of F such that S includes all infinite (i.e., archimedean) prime divisors. If P is a finite (i.e., nonarchimedean) prime divisor we denote by O_P its valuation ring in F. Now, given a finite set S of the above type, we get a ring

$$O_S = \bigcap_{P \notin S} O_P$$

where P ranges over all prime divisors of F. We note in particular that if F is a number field and S the set of infinite prime divisors of F then O_S is just the ring of F-integers.

It is easy to see that there always exists a finite set S satisfying the above hypothesis such that O_S is a unique factorization domain. Hence it seems natural to ask the following two questions:

I. Does there always exist an S such that O_S is a euclidean ring?

II. Can one find an algorithm on O_S for suitably chosen S which is related in some way to the arithmetic of the field F?

The history of the above two questions is as follows: In a series of articles [**1**]–[**4**] Armitage discussed I and II for function fields over arbitrary ground fields. He insisted on a choice of algorithm related to the norm from F to a rational subfield. He showed that if the ground field of F is infinite, then an algorithm of his spacial type was possible if and only if the genus of F is zero. He also discussed the case when the ground field of F is finite, but again the only situation in which he gave a positive answer to I was when F is of genus zero. In [**6**], Samuel also discussed I for function fields F with arbitrary fields of constants, but here also he did not get above genus zero. Finally, in [**5**], M. Madan and the present author showed that the answer to both I and II is yes for function fields of genus one over finite fields. The method in [**5**] was to specifically construct an S and an algorithm on O_S for given F.

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[1] ADDED IN PROOF. After this announcement went to press, the author discovered that Theorem 2 was proved by O. T. O'Meara in *On the finite generation of linear groups over Hasse domains*, J. Reine Angew. Math. **217** (1965), 79–108. MR **31** #3513.