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GALOIS SUBRINGS OF ORE DOMAINS ARE ORE DOMAINS

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If R is a ring, and G is a group of automorphisms of R, then R^G denotes the subring of R consisting of elements of R left fixed by every element of G, and is called the *Galois subring corresponding to* G. In his paper, *Groups acting on hereditary rings*, G. M. Bergman has asked if every Galois subring of a right Ore domain corresponding to a finite group is itself right Ore. In this note we show that the answer is affirmative.

Henceforth, let R denote a right Ore domain with right quotient field D, let G be a finite group of automorphisms, and let $G' = \operatorname{ex} G$ denote the unique extension of G to D. Then, $G' \approx G$ under the restriction map $g' \mapsto g$.

Henceforth, we let G denote a group of automorphisms of D which induces a group of automorphisms of R isomorphic to G under the canonical map. We borrow a term from ring theory coined for another use: the Galois subring R^G will be said to be *right quorite* in case R^G is a right Ore domain with right quotient field $= D^G$. The theorem we prove is slightly stronger than that stated in the title.

THEOREM. *If G is a finite group of automorphisms of a right Ore domain R, then R^G is right quorite.*

Our proof depends heavily on the Cartan-Jacobson Galois theory for division rings, including Jacobson's outer Galois theory of an earlier paper, and concomitant normal basis theorems of Nakayama, Kasch, Tominaga, and the author in the special case when $[D:D^G] = (G:1)$.

Successive reductions for the truth of the theorem can be made to the cases (i) G is an outer or inner group of automorphisms of D (Lemma 1), (ii) G is simple (Lemma 1), (iii) R has prime characteristic p dividing $(G:1)$ (Lemma 2), (iv) G is inner (Lemma 4), and finally, (v) G is cyclic of order p (Lemma 5, ff.).

Both (i) and (ii) are obtained as corollaries of the following lemma:

LEMMA 1. *Assume that there is a subnormal series*

$$G = G_0 \supset G_1 \supset \ldots \supset G_{m-1} \supset G_m = 1. \tag{1}$$

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