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COMPOSITION SERIES AND INTERTWINING OPERATORS FOR THE SPHERICAL PRINCIPAL SERIES

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1. **Introduction.** The purpose of this note is to announce several results (to appear in [4] and [5]) on the spherical principal series for a semisimple Lie group G. Our main result gives a complete determination of the composition series for the case when G has split rank 1. We also give explicit formulas for the intertwining operators for the spherical principal series (for arbitrary rank) and thereby have explicit formulas for parts of the corresponding generalized c-functions. Using the intertwining operators we determine which parts of the composition series of a spherical principal series representation are unitarizable in the case when G has split rank 1.

Except for §5, we exclude the case where $G = SL(2, \boldsymbol{R})$ from the statement of our theorems. For the analogous discussions for $G = SL(2, \boldsymbol{R})$ see Gelfand, Graev, Vilenkin [1] or Sally [9].

2. **Notation and spherical harmonics.** Let G be a real, connected, semisimple, Lie group with finite center. Let $G = KAN$ be an Iwasawa decomposition of G and let M and M' denote respectively the centralizer and normalizer of A in K. Then $W = M'/M$ is the Weyl group of G/K. Let $\mathfrak{g}$, $\mathfrak{k}$, $\mathfrak{a}$, $\mathfrak{m}$, and $\mathfrak{n}$ be respectively the Lie algebras of G, K, A, M and N. If $g \in G$ then $g = k(g)\exp(H(g))n(g)$ where $k(g) \in K$, $H(g) \in \mathfrak{a}$ and $n(g) \in N$ and $k(g)$, $H(g)$ and $n(g)$ are unique.

Let $\mathfrak{a}^*$ be the dual of $\mathfrak{a}$ and $\mathfrak{a}_C^*$ its complexification. If $\gamma \in \mathfrak{a}^* - \{0\}$ then γ is called a restricted root if $g_\gamma = \{X \in \mathfrak{g} \mid [HX] = \gamma(H)X \text{ for all } H \in \mathfrak{a}\}$ is not $\{0\}$. Let $m_\gamma = \dim \mathfrak{g}_\gamma$ and let Δ denote the set of restricted roots. Then $\mathfrak{g} = \sum_{\gamma \in \Delta} \mathfrak{g}_\gamma + \mathfrak{m} + \mathfrak{a}$. We have an ordering on Δ such that if Δ^+ is the set of positive roots then $\mathfrak{n} = \sum_{\alpha \in \Delta^+} \mathfrak{g}_\alpha$. Let $\bar{N}$ be the Lie group corresponding to $\bar{\mathfrak{n}} = \sum_{\alpha \in \Delta^+} \mathfrak{g}_{-\alpha}$. Let $2\rho = \sum_{\alpha \in \Delta} \mathfrak{g} m_\alpha \alpha$.

If $\lambda \in \mathfrak{a}_C^*$ we can define a character on MAN by sending $x = man \to x^\lambda = e^{\lambda(\log a)}$. On $B = G/MAN = K/M$, we fix the unique K-invariant measure db such that $\int_B db = 1$.

Let H^λ be the set of all measurable functions $f: G \to C$ such that
(1) $f(gx) = x^{-\lambda} f(g)$ if $x \in MAN$; and,
(2) $\int_B |f(b)|^2 \, db < \infty$.

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