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ON THE HAEFLIGER KNOT GROUPS

BY R. JAMES MILGRAM[1]

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ABSTRACT. Let C_n^k be the group of isotopy classes of differentiable embeddings $S^n \subset S^{n+k}$. In [4], A. Haefliger established an isomorphism $C_n^k \cong \pi_{n+1}(G, SO, G_k)$ where G_k is the set of (oriented) homotopy equivalences of the sphere S^{k-1}. In this note, we indicate methods which make the calculation of these groups feasible. In particular, we determine the first seven nonzero groups in the metastable range. We also develop connections between the composition operation in homotopy and such geometric operations as spin-twisting knots.

1. The space of the Haefliger knot groups.

THEOREM A. *There is a space F_k which is $2k-3$-connected and $\pi_n(F_k) \cong C_n^k$.*

Indeed, F_k is the fiber in the map

$$SG_k/SO_k \to SG/SO,$$

induced by the usual inclusions $B_{G_k} \hookrightarrow B_G$, $B_{SO_k} \hookrightarrow B_{SO}$. Alternately, F_k is the fiber in the map

$$SO/SO_k \to G/G_k,$$

induced by the inclusions $B_{SO_k} \hookrightarrow B_{G_k}$, $B_{SO} \hookrightarrow B_G$.

COROLLARY B. *$H_*(F_k; Z_2) \cong E(\cdots A_I \cdots)$ where I runs over all sequences of integers $(i_1, \ldots, i_t)$, satisfying*

(i) $0 \leqq i_1 \leqq \ldots \leqq i_t$ $(t \geqq 2)$,
(ii) $i_1 = 0$ *implies* $t = 2$,
(iii) $i_t \geqq k-1$.

Moreover, $\dim(A_I)$ *is* $i_1 + 2i_2 + 4i_3 + \ldots + 2^{t-1}i_t - 1$. *(Here E is an exterior algebra on these stated generators.)*

B follows from A on applying the results of [7]. In the same way, it is possible to obtain partial information about $H_*(F_k; Z_p)$ for p odd. Similarly, we can determine $H^*(F_k; Z_2)$ as a module over the Steenrod algebra $\mathscr{A}(2)$, and $H^*(F_k; Z_p)$ over $\mathscr{A}(p)$ in the range of dimensions less than $3k-2$.

In this range, $H^*(F_k; Z_p)$ has one nonzero generator e_{4s-1} in each dimension $4s-1$, and is zero otherwise. For general p, the $\mathscr{A}(p)$-structure

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