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THE DUAL GROUP OF THE FOURIER-STIELTJES ALGEBRA

BY MARTIN E. WALTER[1]

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ABSTRACT. We announce the definition of the dual group, $G_{B(G)}$, of the Fourier-Stieltjes algebra, $B(G)$, of a locally compact group G; and we state four main theorems culminating in the result that $G_{B(G)}$ is a locally compact topological group which is topologically isomorphic to G. This result establishes an *explicit* dual relationship between a group and its Fourier-Stieltjes algebra. Moreover, this result extends naturally the notion of Pontriagin duality to the case of noncommutative groups.

1. We shall adopt the notation and assume familiarity with the results of [3], [4]. We recall from [3], [4] that each $\phi \in \mathrm{Aut}(B(G))$, the isometric algebra automorphisms of the Banach algebra $B(G)$, can be written in the form

$$\phi = {}^tT_g{}^t\psi \tag{1}$$

where $\langle x, {}^tT_g b\rangle = \langle gx, b\rangle$ and $\langle x, {}^t\psi b\rangle = \langle \psi(x), b\rangle$ for all $b \in B(G)$ and all $x \in G$, with $g \in G$ and $T_g : x \in G \mapsto gx \in G$, a left translation of G, and $\psi \in \mathrm{AAut}(G)$, the set of all topological automorphisms and/or anti-automorphisms of G. The set of all topological automorphisms of G is denoted by $\mathrm{Aut}(G)$. Note that for a given $\phi \in \mathrm{Aut}(B(G))$ decomposition (1) is unique; and recall that superscript ^{t} indicates the transpose in the appropriate duality. We let $I_g : x \in G \mapsto g^{-1}xg \in G$ denote the inner automorphism of G induced by g; and we observe that $T_gI_g : x \in G \mapsto xg \in G$ is a right translation of G.

2. We begin by defining the underlying topological space of (what shall eventually be the dual group of $B(G)$) $G_{B(G)}$. This requires first the following *intrinsic characterization* of $A(G)$, the Fourier algebra of G, within $B(G)$.

THEOREM 1. *The Fourier algebra of G, $A(G)$, is the smallest norm-closed, nonzero ideal in $B(G)$ which is invariant under* $\mathrm{Aut}(B(G))$, *i.e.*,

$$A(G) = \bigcap \{ \mathcal{I} : \mathcal{I} \text{ is a norm-closed, nonzero ideal in } B(G) \text{ such that } \phi(\mathcal{I}) \subset \mathcal{I} \text{ for all } \phi \in \mathrm{Aut}(B(G))\}.$$

REMARK. The burden of the proof rests on the generalized tauberian

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