BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 78, Number 5, September 1972

A GENERALIZATION OF THE EILENBERG-MOORE SPECTRAL SEQUENCE

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Communicated by E. H. Spanier, February 24, 1972

The purpose of this note is to announce a generalization of the (cobar) spectral sequence of Eilenberg and Moore. Full proofs and further applications will be given elsewhere.

In this paper, "space" will mean simplical set, though this is not essential to our arguments, and topological spaces could be used with some modifications.

Suppose that X is a cosimplicial space. If K is a field, let $K(X)$ be the cosimplicial simplicial vector space which is obtained by taking the degreewise free K-vector space on the sets $X[m](n)$ (the square brackets refer to cosimplicial degree, the parentheses to simplicial degree). There is a cosimplicial map $X \to K(X)$ which is the extension of the set map which sends each element into the basis vector which it determines. If $|\ |$ denotes the "geometric (co)realization" in the sense of Bousfield and Kan [**1**], there is a realization $|X| \to |K(X)|$ of this map. Since $|K(X)|$ is easily seen to be a K-vector space, we see that there is an obvious extension $K(|X|) \to |K(X)|$ which is K-linear. Our main theorem will state that under certain circumstances, this map will be a weak equivalence of simplicial groups. Thus, $H_*(|X|; K) = \pi_* K(|X|) = \pi_*|K(X)|$. There is a standard spectral sequence for $\pi_*|K(X)|$, for which $E^1_{p,q} = H_p(X[q]; K)$.

In order to see that this spectral sequence includes the usual Eilenberg-Moore spectral sequence, suppose that we have a pair of maps $A \to C \leftarrow B$ of Kan complexes. Let $X[0] = A \times B$, and let $X[n] = X[0] \times C^n$ (in Rector's [**4**] terms, X is the geometric cobar construction $\mathscr{B}^C(A, B)$). The coface maps are given by diagonals and by the maps $A \to C \leftarrow B$, codegeneracies by projections. Then if $D = |X|, D$ is easily seen to be the homotopy fiber product of A and B over C. Since $E^1_{p,*} = H_*(A) \otimes H_*(C^p) \otimes H_*(B) = \mathscr{B}_p^{H_*(C)}(H_*(A), H_*(B))$, the cobar construction on $H_*(A)$ and $H_*(B)$ over $H_*(C)$, we see that $E^2_{p,q} = \operatorname{Cotor}^{H_*(C)}_{p,q}(H_*(A), H_*(B))$ (all homology groups with K-coefficients).

A second application of this spectral sequence is the computation of the homology of function spaces Y^Z. In order for our convergence conditions to hold, we need that Y be a Kan complex, and that the connectivity of Y

AMS 1970 *subject classifications*. Primary 55H20.