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BALANCED LOCAL RINGS WITH COMMUTATIVE RESIDUE FIELDS

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1. Let R be a ring with unity. An R-module M is said to be balanced or to have the double centralizer property, if the natural homomorphism from R to the double centralizer of M is surjective. If all left and right R-modules are balanced, R is called *balanced.* It is well known that every artinian uniserial ring is balanced. In [5], J. P. Jans conjectured that those were the only (artinian) balanced rings. Jans' conjecture has been shown to be false in [3] by constructing a class of balanced nonuniserial rings. In the present paper, we show that the rings of [3] together with the (local) uniserial rings are the only balanced rings which are local (i.e. have a unique one-sided maximal ideal) and whose residue division ring R/W is commutative (here, as well as in what follows, W denotes always the radical of R).

THEOREM. *Let R be a local ring with the radical W, and let $Q = R/W$ be commutative. Then R is balanced if and only if either*

(a) *R is an artinian uniserial ring, or*

(b) $W^2 = 0$ *and* $\dim({}_QW) \cdot \dim(W_Q) = 2$.

In view of [2] combined with [4], an arbitrary (not necessarily local) balanced ring is a finite direct sum of full matrix rings over balanced local rings, and thus we get the following:

COROLLARY. *Let R be a ring with the radical W such that R/W is commutative. Then R is balanced if and only if R is a finite direct sum of local rings of type* (a) *or* (b) *of Theorem.*

The proof of Theorem is based on the structure theorems of [2] and a result of V. P. Camillo and K. R. Fuller [1] on the index of a certain division subring of R/W. More specifically, the following two theorems of [2] will be used in our arguments (in [2], the formulations of these theorems are more general).

THEOREM A* OF [2]. *A balanced ring is artinian.*

THEOREM B* OF [2]. *Let R be a balanced local ring with the radical W; put $Q = R/W$. Then either*

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