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THE FUNDAMENTAL FORM OF A FINITE PURELY INSEPARABLE FIELD EXTENSION

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The purpose of this note is to show that to every finite purely inseparable field extension K/k there is associated in a natural way a symmetric cochain $f:K \times \ldots \times K$ (γ times) $\to K$ of K with coefficients in itself which we call the "fundamental form" of K. Its degree, γ, depends on certain structural properties of K. The fundamental form is a derivation when considered as a function of any one variable, all others being held fixed. (It is almost always a coboundary when viewed as a function of all variables.) If K is a tensor product of two intermediate fields then its fundamental form is a certain symmetric product of the forms of the intermediate fields. A weak converse is known and a strong one conjectured.

References in this note to Nakai are to [**4**] and [**5**], those to Keith are to [**3**].

1. DEFINITION. Let A be a commutative k-algebra, set $Y^1(A) = \operatorname{End}_k A$ and for every $n > 1$ let $Y^n(A) = Y^n$ be the set of those n-cochains f of A with coefficients in itself which are symmetric as functions of all n variables and which have the property that if all but two variables are fixed then f is a two-cocycle when considered as a function of the remaining ones. If $f \in Y^n$, then the $n+1$-cochain Δf defined by $\Delta f(a_1, \ldots, a_{n+1}) = a_n f(a_1, \ldots, a_{n-1}, a_{n+1}) - f(a_1, \ldots, a_{n-1}, a_n a_{n+1}) + a_{n+1} f(a_1, \ldots, a_n)$ is in Y^{n+1}. This defines the "Nakai operator" $\Delta: Y^n \to Y^{n+1}$. It is easy to verify that for odd n, Δ is identical with the Hochschild coboundary operator δ restricted to Y^n. However, in general, $\Delta^2 \neq 0$ and the Y^i do not form a complex. Those elements of Y^1 which are annihilated by Δ^i are called "ith order derivations" or simply "i-derivations" and form an A-module denoted by $\mathscr{D}^i$. A 1-derivation is an ordinary derivation of A into itself. If A is unital, which we henceforth assume, then we denote by Y_0^n the submodule of Y^n consisting of those cochains in Y^n which vanish when any variable equals 1. We then have $\Delta Y_0^n \subset Y_0^{n+1}$, and $\mathscr{D}^i \subset Y_0^1$ for all i. If $\varphi \in \mathscr{D}^i$, $\psi \in \mathscr{D}^j$ then their composite $\varphi\psi$ is in $\mathscr{D}^{i+j}$ (Nakai). The space $\bigcup_{i=1}^{\infty} \mathscr{D}^i$ of all "high order derivations" is thus a ring with an increasing filtration. When A is a finite purely inseparable field extension

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