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CLASSIFICATION THEOREMS FOR p-GROUPS AND MODULES OVER A DISCRETE VALUATION RING

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In 1933, Ulm gave a complete classification of countable, Abelian p-groups in terms of a family of additive, numerical invariants. Recently, R. Nunke discovered a large class of p-groups, the totally projective groups, which is a natural extension of the class of countable groups. Ulm's theorem was generalized to this class by P. Hill. The same class was discovered independently, from the point of view of generators and relations, by Crawley and Hales. In this note we announce three generalizations of this theory. We introduce two new classes of mixed modules over a discrete valuation ring, and a new class of p-groups (containing the totally projective groups). In each case, we also introduce a new family of invariants which can be used, together with Ulm's invariants, to prove a classification theorem.

In the following, $\boldsymbol{R}$ will be a discrete valuation ring, R will be the ring regarded as a module over itself, and p will be a generator of the maximal ideal of $\boldsymbol{R}$. If M is any $\boldsymbol{R}$-module, we define $p^\alpha M$ for any ordinal α by $p^{\alpha+1}M = p(p^\alpha M)$ and, if α is a limit ordinal, $p^\alpha M = \bigcap_{\beta<\alpha} p^\beta M$. We recall that a module M is *reduced* if it has no nonzero divisible submodules, or, equivalently, if for some ordinal α, $p^\alpha M = 0$. A module is a *T-module* [**1**] if it can be given in terms of generators and relations in such a way that all of the relations have the form $px = y$ or $px = 0$. A torsion module M is *totally projective* [7] if for any module N and any ordinal α, $p^\alpha \operatorname{Ext}(M/p^\alpha M, N) = 0$. The work of Nunke [7], Crawley and Hales [**1**], and Hill [**2**], suitably generalized to modules over a discrete valuation ring, shows that a torsion, reduced module is a T-module if and only if it is totally projective, and that these modules can be classified in terms of their Ulm invariants (defined below).

We will call a class $\mathscr{C}$ of R-modules *closed* if it satisfies the following conditions:

(i) A module isomorphic to a module in $\mathscr{C}$ is in $\mathscr{C}$.

(ii) A direct sum of modules in $\mathscr{C}$ is in $\mathscr{C}$.

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