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QUASI-ANALYTICITY AND SEMIGROUPS OF BOUNDED LINEAR TRANSFORMATIONS

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Suppose H is a real Banach space and T is a strongly continuous (on $[0, \infty)$) semigroup of bounded linear transformations from H to H. Steps leading to the following will be indicated:

THEOREM. *If*

$$\liminf_{x \to 0} |T(x) - I| < 2 \tag{1}$$

then the set of all functionals of trajectories of T form a quasi-analytic collection.

COROLLARY. *If* (1) *is satisfied, then $T(x)$ is invertible for all $x > 0$ (although $(T(x))^{-1}$ may be unbounded).*

A functional of a trajectory of T is a function h with domain $(0, \infty)$ for which there is f in H^* and p in H so that $h(x) = f(T(x)p)$ for all $x > 0$. A collection G of real-valued functions with a common connected domain J is quasi-analytic provided no two members of G agree on an open subset of J.

In [7] it is shown that if

$$\limsup_{x \to 0} |T(x) - I| < 2, \tag{2}$$

then every functional of a trajectory of T is real-analytic (and $AT(x)$ is bounded for all $x > 0$ where A is the generator of T). An example in [7] can be used to show that (1) does not imply (2).

Recent closely related results [**1**], [**3**], [**8**], [**9**], [**2**] as well as [**7**] connect the following: (a) the degree of approximation of the identity by the semigroup, (b) properties of the generator and (c) regularity properties of trajectories. For T a Markov semigroup it may be seen from [**4**] that (1) follows from a condition on transition probabilities ($\Gamma > 0$).

Lemmas 1 and 2 which follow are improvements of Lemma 7 of [**6**] and Theorem 1 of [**5**] respectively.

Suppose f is a real-valued continuous function with domain $[0, 1]$ so that $f(x) = 0$ if $0 \leq x \leq \frac{1}{2}$ and, if $y > \frac{1}{2}$, then there is a number x in $(\frac{1}{2}, y)$

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