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JORDAN OPERATORS IN INFINITE DIMENSIONS AND STURM LIOUVILLE CONJUGATE POINT THEORY[1]

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This note concerns the two simplest types of bounded operators with real spectrum on a Hilbert space H. The purpose of this note is to suggest an abstract algebraic characterization for these operators and to point out a rather unexpected connection between such algebraic considerations and the classical theory of ordinary differential equations. In particular, our Theorem II which gives an algebraic characterization of certain subjordan operators (defined below) seems very closely related to the classical theorem asserting that a Sturm Liouville operator defined on the interval $[a, b]$ is positive definite if and only if there are no points conjugate to a in the interval. One appealing thing is that almost every idea presented here has a natural generalization worthy of investigation.

The two types of operators considered here are:

Jordan operators (order k)—operators of the form $S + N$ where S is selfadjoint, S commutes with N, and $N^k = 0$.

Subjordan operators (order k)—operators which are unitarily equivalent to the restriction of a Jordan operator C to an invariant subspace of C.

A natural algebraic condition on a bounded operator T which generalizes the selfadjointness condition is

POL n
$$e^{-isT^*}e^{isT} = \sum_{k=0}^{n} A_k s^k.$$

This is equivalent to

$$\frac{d^{n+1}}{ds^{n+1}} e^{-isT^*}e^{isT} = 0$$

which is in turn equivalent to $C_T^{n+1}(I) = 0$, where $C_T : \mathscr{L}(H) \to \mathscr{L}(H)$ is defined by $C_T(A) = T^*A - AT$ and C_T^k denotes the composition of the map C_T with itself k times. An operator T which satisfies POL n will be called *coadjoint* (order n). Note that if T is coadjoint, then T^* is not necessarily coadjoint. The results announced here concern coadjoint operators of the second order only. Details of proofs will appear elsewhere.

THEOREM I. *An operator T is Jordan* (*order* 2) *if and only if both T and T^* are coadjoint* (*order* 2).

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