BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 77, Number 6, November 1971

DEFORMATIONS OF NORMAL VECTOR FIELDS AND THE GENERALIZED MINKOWSKI PROBLEM

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Communicated by M. H. Protter, June 7, 1971

In this note I announce a solution to the

GENERALIZED MINKOWSKI PROBLEM. *Find an embedding of the m-sphere S^m into Euclidean space R^{m+1}, whose Gaussian curvature is preassigned as a continuous, strictly positive function on S^m.*

Such embeddings are shown to exist without exception for $m \geqq 2$, but with certain necessary exceptions for $m=1$, the result in this case standing as a converse to the classical Four Vertex Theorem for plane curves.

While the above problem lies in the realm of differential geometry in the large, its solution comes via differential topology by studying deformations of normal vector fields on a closed smooth manifold M^m in R^{m+1}.

Detailed proofs may be found in [9], while a leisurely exposition of the converse to the Four Vertex Theorem appears in [10]. I am indebted to Eugenio Calabi, Jerry Kazdan and Frank Warner for many helpful conversations, and particularly to Warner for introducing me to the problem in the first place and for pointing out its relation to the study of normal vector fields on spheres.

1. **The generalized Minkowski problem.** In the classical Minkowski problem, one starts only with those continuous strictly positive curvature functions $K: S^m \to R^1$ which satisfy the integrability condition $\int_{S^m} N(p)/K(p)\, d\Omega = 0$, where $N(p)$ is the unit outward normal vector to the unit m-sphere S^m in R^{m+1}. In return, these preassigned curvatures are realized not by arbitrary embeddings of S^m into R^{m+1}, but by inverses of Gauss maps, and one also gets a corresponding uniqueness theorem up to parallel translations. The reader can find an historical survey of this problem in [9]; the original sources are Minkowski [13], [14], Bonnesen and Fenchel [6], Lewy [12],

AMS 1970 *subject classifications*. Primary 53C45, 53C20, 57D25; Secondary 57D50.
Key words and phrases. Minkowski problem, Four Vertex Theorem, Gaussian curvature, Riemannian metric, convex surface, convex hypersurface, vector field, normal vector field, deformation.
[1] The author acknowledges partial support from the National Science Foundation grant GP-19693.