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A NOTE ON MACKEY'S IMPRIMITIVITY THEOREM

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The imprimitivity theorem—Theorem 2 in [M1]—has many applications, in particular, to quantum mechanics (cf. [M2]). It can be paraphrased as follows:

Every imprimitive (cf. [M1]) unitary representation of a separable locally compact group G is unitarily equivalent to an induced representation.

The importance of this theorem is further reflected in the fact that Fell devotes a sizable part of his recent book [F1] to a generalization of this theorem.

Blattner succeeded in [B1] and [B2] to remove the separability assumption on G and at the same time to simplify the proof of this theorem. In this note we observe how some of the intricacy of Blattner's proof can be reduced to an application of Fubini's theorem.

We are using the same notation as in [B2]. Thus, H is a closed subgroup of the locally compact group G, π is the quotient map from G onto the quotient space G/H on which G acts from the right, and $d\xi$ and dx denote (right) Haar measure on H and on G with modular functions δ_H and δ_G. The measure μ on $G\times G$ implicitly defined in Lemma 2 of [B2] can be explicitly defined for k in $C_0(G\times G)$ by

$$\begin{aligned}(1)\qquad &\iint k(y,z)\,d\mu(y,z)\\ &\qquad = \iiint k(zy^{-1}\xi^{-1}, y^{-1}\xi^{-1})\delta_G^{-1}(\xi y)\,d\xi\,d\Lambda(\pi(y),z).\end{aligned}$$

The key formula, in Blattner's proof of Theorem 2,

$$\int h(x)(f(x),g(x))_\mu\,dx = (P(\tau h)f,f)_\Lambda, \tag{2}$$

can now be directly derived as follows: using the definitions of $(\cdot,\cdot)_\mu$, $\hat{f}$ and (1) the left-hand side of (2) becomes

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