BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 77, Number 6, November 1971

WEIGHTED APPROXIMATION OF CONTINUOUS FUNCTIONS[1]

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Communicated by Felix Browder, June 17, 1971

1. **Notation.** Let X be a completely regular Hausdorff space and E a (real or complex) locally convex Hausdorff space. $F(X, E)$ is the vector space of all mappings from X into E, and $C(X, E)$ is the vector subspace of all such mappings that are continuous. $B_\infty(X, E)$ is the vector subspace of $F(X, E)$ consisting of those bounded f that vanish at infinity. The vector subspace $C(X, E) \cap B_\infty(X, E)$ is denoted by $C_\infty(X, E)$. If X is locally compact, $\mathcal{K}(X, E)$ will denote the subspace of $C(X, E)$ consisting of those functions that have compact support. The corresponding spaces for $E = \boldsymbol{R}$ or $\boldsymbol{C}$ are written omitting E. A *weight* v on X is a nonnegative upper semicontinuous function on X. A directed family of weights on X is a set of weights on X such that given $u, v \in V$ and $\lambda \geqq 0$ there is a $w \in W$ such that $\lambda u, \lambda v \leqq w$. If U and V are two directed families of weights on X and for every $u \in U$ there is a $v \in V$ such that $u \leqq v$, we write $U \leqq V$. If V is a directed family of weights on X, the vector space of all $f \in F(X, E)$ such that $vf \in B_\infty(X, E)$, for any $v \in V$, is denoted by $FV_\infty(X, E)$ and is called a weighted function space. On $FV_\infty(X, E)$ we shall consider the topology determined by all the seminorms $f \mapsto \sup \{v(x)p(f(x)) ; x \in X\}$ where $v \in V$ and p is a continuous seminorm on E. $CV_\infty(X, E)$ will denote the subspace $FV_\infty(X, E) \cap C(X, E)$, equipped with the induced topology. The weighted function spaces $CV_\infty(X, E)$ will be called *Nachbin spaces.*

2. **Completeness properties of Nachbin spaces** [6]. If for every $x \in X$ there is a weight $u \in U$ such that $u(x) > 0$, we write $U > 0$.

LEMMA. *If E is complete and $U > 0$, then $FU_\infty(X, E)$ is complete.*

THEOREM 1. *Suppose that E is complete, and U and V are two directed families of weights on X with $U \leqq V$. If $V > 0$ on X and $CU_\infty(X, E)$ is closed in $FU_\infty(X, E)$, the Nachbin space $CV_\infty(X, E)$ is complete.*

AMS 1970 *subject classifications.* Primary 46E10, 46E40; Secondary 46G10.

Key words and phrases. Continuous vector-valued functions, weights, Nachbin spaces, completeness, vector-valued bounded Radon measures, Bishop's generalized Stone-Weierstrass theorem, tensor product of topological modules.

[1] This research was supported in part by NSF Grant GP-22713.