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# INVARIANTS FOR SEMIFREE $S^1$-ACTIONS

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1. **Introduction.** In [1], Atiyah and Singer obtained an invariant for certain $S^1$-actions and in [5], Browder and Petrie used the invariant to distinguish certain semifree $S^1$-actions so that they showed the following result.

THEOREM 0. *For any odd* $n \geqq 5$, $n = 2k-1$, *there are an infinite number of distinct semifree* $S^1$-*actions on the Brieskorn* $2n+1$ *spheres with fixed point set of codimension* $4m$, *any* $m \neq k/2$, $m < (n-1)/2$.

The purpose of the present paper is to define some invariants for certain semifree $S^1$-actions which are different from that of Atiyah and Singer (see Theorem 1). As an application, we can prove the above theorem of Browder and Petrie without the assumption $m \neq k/2$ (see Corollary 2). Our method is different from that of Atiyah and Singer, and Browder and Petrie. We use the Chern classes due to Borel and Hirzebruch [2] and Grothendieck (see [3]) and the bordism theory. The author wishes to thank Professor F. Uchida who kindly enlightened him about the structure of the normal bundle of fixed point set.

Detailed proof will appear elsewhere.

2. **Definitions and statement of results.**

DEFINITIONS. An action $(M, \phi, G)$ is called semifree if $G$ acts freely outside the fixed point set.

Denote by $B(\xi)$, $S(\xi)$ and $\boldsymbol{C}P(\xi)$, the total space of the disk bundle, the total space of the sphere bundle and the total space of the projective space bundle respectively associated to a complex vector bundle $\xi$. We remark the following. Given a semifree $S^1$-action $(M, \phi, S^1)$ where $M$ may have a boundary, the normal vector bundle of the fixed point set has the unique complex vector bundle structure such that the induced action of $S^1$ is the scalar multiplication when we regard $S^1$ as $\{z \mid z \in \boldsymbol{C}, |z| = 1\}$. By "complex bundle" we mean that such a complex structure is taken. Let $(M, \phi, S^1)$ be a semifree $S^1$-action

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