BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 77, Number 6, November 1971

SOME NEW RESULTS IN THE KOLMOGOROV-SINAI THEORY OF ENTROPY AND ERGODIC THEORY[1]

BY DONALD S. ORNSTEIN

Introduction. Origins of ergodic theory. Our results will be concerned with 1-1 invertible transformations, T, of a measure space X, of total measure 1, where T and T^{-1} transform each measurable set onto a measurable set of the same measure.

We will be concerned with the behavior of T only up to sets of measure 0. That is, we will say that T_1 acting on X_1 is isomorphic to T_2 acting on X_2 if we can find $\overline{X}_1 \subset X_1$ and $\overline{X}_2 \subset X_2$ of measure 1 and invariant under T_1 and T_2, respectively, and if there is an invertible, measure-preserving transformation ψ of $\overline{X}_1$ onto $\overline{X}_2$ such that for all x in $\overline{X}_1$ we have $\psi T_1(x) = T_2\psi(x)$.

Measure-preserving transformations arise in many different contexts, and I will now describe three of the more important ones.

(1) *Automorphisms of compact groups.* Any continuous automorphism of a compact group is a measure-preserving transformation with respect to Haar measure (see Halmos, *Lectures in ergodic theory*).

(2) *Random processes.* A stationary process can be thought of as a box that prints out one letter for each unit of time where the probability of a given letter being printed out may depend on the letters printed out in the past but is independent of the time (that is, the mechanism in the box does not change).

EXAMPLE 1. The box contains a roulette wheel. We spin the wheel once each unit of time and print out the result.

EXAMPLE 2. The box contains a roulette wheel. We look at all possibilities for three consecutive spins and divide these into two classes. Now each time we spin the wheel we look at the last three spins and print out a 1 if they fall in the first class and a 2 if they fall in the second class.

EXAMPLE 3. A teleprinter. This prints out letters where the probability of a given letter depends on what has already been printed (many possibilities will have probability 0 because they will not make sense).

An Invited Address originally presented under the title *The isomorphism problem in ergodic theory* at the 680th meeting of the Society in Pasadena, California, November 21, 1970; received by the editors May 21, 1971.

AMS 1970 *subject classifications.* Primary 28A65; Secondary 54H20.

[1] This research was supported in part by NSF grant GP 21509.