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HOLOMORPHIC APPROXIMATION ON TOTALLY REAL SUBMANIFOLDS OF A COMPLEX MANIFOLD[1]

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1. **Introduction.** Let M be a real differentiable submanifold of a complex manifold X of differentiability class C^k, $1 \leqq k \leqq \infty$. Letting $T_x(M)$ and $T_x(X)$ denote the tangent spaces to M and X at $x \in M$, we see that $T_x(M)$ is a real linear subspace of the complex vector space $T_x(X)$. We say that M is a *totally real submanifold* of X if for each point $x \in M$, $T_x(M)$ contains no nonzero complex subspaces of $T_x(X)$.

There are many examples of totally real submanifolds (see, e.g. Nirenberg-Wells [8]), the simplest examples being a real curve in X, the distinguished boundary of a polydomain, or $\boldsymbol{R}^n \subset \boldsymbol{C}^n$. The geometric nature of a totally real submanifold implies that it behaves in many cases like this last example. In particular, the Weierstrass approximation theorem tells us that holomorphic functions on $\boldsymbol{C}^n$ are dense in the Banach space of continuous functions on a compact subset $K \subset \boldsymbol{R}^n \subset \boldsymbol{C}^n$ in the supremum norm. There have been various investigations recently generalizing this type of theorem to compact subsets of totally real submanifolds (see Čirka [1], Hörmander-Wermer [6], Nirenberg-Wells [8]). In §2 we formulate our main results on holomorphic approximation, in which we improve on the previous known results by (a) extending the domain of definition of the approximating functions, (b) minimizing the differentiability requirements for the submanifold, and (c) requiring that the approximation be uniform on K along with uniform approximation of all derivatives up to the order of differentiability of the submanifold. In §3 we formulate sheaf injection theorems for hyperfunctions on a totally real submanifold of a complex manifold. For instance, Theorem 4.1 says that the sheaf of germs of distributions is canonically embedded in the sheaf of germs of hyperfunctions on a C^∞ totally real submanifold of a complex manifold (cf. Martineau [7] and Harvey [4]).

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