BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 77, Number 5, September 1971

VARIETIES OF LOCALLY CONVEX TOPOLOGICAL VECTOR SPACES

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Communicated by Victor Klee, April 14, 1971

1. **Introduction.** A variety is defined here to be a nonempty class of real Hausdorff locally convex spaces (LCS's) closed under the operations of taking subspaces (not necessarily closed), separated quotients, arbitrary products and isomorphic images. The two extreme examples of a variety are the class of all LCS's and the class of all zero-dimensional LCS's. Less obvious examples are:

(a) the class of all Schwartz LCS's [8],
(b) the class of all nuclear LCS's [26],
(c) the class of all LCS's having their weak topology [8].

The potency of an analogous concept for groups [21] has manifested itself for three decades, and for topological groups has quite recently been asserted [3], [14]–[20]. In this note we announce selected results from a forthcoming paper [5] which, we hope, will convince the reader that the theory of varieties not only is of intrinsic interest, but lends to locally convex spaces a new and illuminating perspective which consolidates, strengthens and adds to significant parts of the literature.

2. **Results.** For any class $\mathcal{C}$ of LCS's, the *variety generated by* $\mathcal{C}$, denoted by $\mathcal{V}(\mathcal{C})$, is the smallest variety containing $\mathcal{C}$. For example, the variety generated by the class of all real Banach spaces is the class of all LCS's [26].

Consider these seven properties for Banach spaces: (i) reflexivity; (ii) quasi-reflexivity [4]; (iii) almost reflexivity [10]; (iv) separability; (v) having separable dual; (vi) being Hilbertian; (vii) having Hamel dimension $<\aleph$, where $\aleph$ is some fixed infinite cardinal. Theorem 1 is a stronger statement than the usual ones about closed subspaces, separated quotients and finite products of Banach spaces with one of the above properties.

AMS 1970 *subject classifications.* Primary 46A05; Secondary 46E10, 46E30, 46A45, 46M99.

Key words and phrases. Products, subspaces and quotients of locally convex spaces, Banach spaces, Fréchet spaces, nuclear spaces, Schwartz spaces, reflexivity, separability, weak topology, strongest locally convex topology, $l_1(\Gamma)$, l_p, L_p, c_0, spaces of continuous functions, singly generated variety, universal generator.