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PARTIAL DIFFERENTIAL EQUATIONS IN FISCHER-FOCK SPACES FOR THE HILBERT-SCHMIDT HOLOMORPHY TYPE

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1. **Introduction.** Current work on the extension of function theory to infinite-dimensional domains has led to the consideration of classes of analytic functions defined on Banach spaces, with Fréchet derivatives of a given type, e.g., nuclear, compact or integral. The existence theory of partial differential equations in this setting follows from [G] for the nuclear bounded case, and is given in [D] for formal power series of α-β-γ-type. In this note we describe the duality theory (Theorem 1) and the existence theory (Theorem 2) of partial differential equations for a class of spaces of entire functions defined on a Hilbert space, with Fréchet derivatives given by Hilbert-Schmidt operators. When the underlying Hilbert space is finite-dimensional, we recover results in [T, Chapter 9], in [B] and in [NS] (Fischer space). When the underlying space is a Hilbert space of square-integrable functions, we obtain the wave functionals in the Fock representation of quantum field theory (cf. [NT]), subsuming some of the results proved independently in [R].

2. **Hilbert-Schmidt polynomials.** Let E be a Hilbert space over the complex field $\mathbf{C}$, with inner product $(\ |\ )$, and E' the dual of E, with the inner product $(u'|v') = (v|u)$ for $u' = (\ |u)$, $v' = (\ |v)$. Let $E'^{\vee n} = E' \vee \cdots \vee E'$ denote the n-fold symmetric product of E' [Gr, p. 191]. The Hilbert-Schmidt inner product on $E'^{\vee n}$ is characterized for decomposable elements by

$$(u'_1 \vee \cdots \vee u'_n \,|\, v'_1 \vee \cdots \vee v'_n) = \frac{1}{n!} \sum_{\pi} (u'_{\pi 1} | v'_1) \cdots (u'_{\pi n} | v'_n),$$

the summation extended over all permutations π of the indices. $E'^{\vee n}_H$ denotes the n-fold symmetric product equipped with the Hilbert-

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