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# A MAXIMUM PRINCIPLE FOR OPTIMAL CONTROL PROBLEMS WITH NEUTRAL FUNCTIONAL DIFFERENTIAL SYSTEMS[1]

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We present a maximum principle in integral form for optimal control problems whose system equations involve delays in the state and delays in the derivative of the state. The results are obtained for a very general class of neutral functional differential equations which includes as a special case the systems

$$\dot{x}(t) + A\dot{x}(t-h) = Bx(t) + Cx(t-h) + Du(t),$$

which have been studied extensively (as in [4]) and arise in many applications. The class of control problems considered include problems for which one wishes to minimize $\int_0^T x^2(t)\,dt$ while requiring that $u(t) \in U \subset R^n$, $t \in [0, T]$, and either $x|_{[T-h,T]}$ lie in a manifold in $AC([T-h, T], R^n)$ or $x(t) = \zeta(t)$ on $[T-h, T]$, $\zeta$ a fixed absolutely continuous function. These functional boundary conditions arise naturally since the "state" in neutral systems of the above type is a point in $AC([-h, 0], R^n)$.

Let $\alpha_0$, $t_0$, and $a$ be fixed in $R$ with $-\infty < \alpha_0 < t_0 < a < \infty$, $I = [\alpha_0, a)$, $I' = [t_0, a)$. For $x$ continuous on $I$ and $t$ in $I'$, the notation $F(x(\cdot), t)$ will mean $F$ is a functional in $x$, depending on any or all of the values $x(\tau)$, $\alpha_0 \leqq \tau \leqq t$. For $t \in I'$, let

$$D(x(\cdot), t) = x(t) - \sum_{l=1}^{p} a_l(t)x(h_l(t)) - \int_{\alpha_0}^{t} d_\theta[\nu(t, \theta)]x(\theta).$$

Assume $a_l: I' \to R^{n^2}$ is continuous and of bounded variation, $l = 1, \cdots, p$; $h_l: I' \to R$ is continuous and strictly increasing, and there exists $\Delta > 0$ such that $\alpha_0 \leqq h_l(t) < t - \Delta$, $l = 1, \cdots, p$. Let $\nu(s, \cdot): [\alpha_0, \infty)$

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