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TOEPLITZ OPERATORS IN MULTIPLY CONNECTED REGIONS

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1. **Introduction.** Let D be a bounded domain in the plane whose boundary consists of $n+1$ nonintersecting, analytic, Jordan curves. Let C be the space of continuous complex functions on ∂D and let A be the subspace of C consisting of those functions with continuous extensions to $\bar{D}$ which are analytic in D. Let m be harmonic measure on ∂D for some point in D and let H^2 be the $L^2(m)$-closure of A. Let P denote the orthogonal projection of $L^2(m)$ onto H^2. For ϕ in $L^\infty(m)$ the Toeplitz operator T_ϕ on H^2 is defined by $T_\phi(f)=P(\phi f)$ for f in H^2. This paper deals with the Fredholm theory and with invertibility criteria for these operators.

On the Fredholm level, these Toeplitz operators are similar to the usual Toeplitz operators on the disk (cf. [3]). For instance, if ϕ is in C, then the essential spectrum of T_ϕ is the range of ϕ (Theorem 3). However, the spectrum itself may be considerably different: in this setting there are selfadjoint Toeplitz operators with eigenvalues and disconnected spectrum.

This note is primarily a statement of results and discussion. Complete proofs will appear elsewhere. The results are from my dissertation written under the direction of R. G. Douglas.

2. **The algebra H^∞.** Let H^∞ be the weak star closure of A in $L^\infty(m)$. The following theorem is due to Tumarkin and Havinson [10] and is generalized to hypo-Dirichlet algebras by Ahern and Sarason [1].

THEOREM. *If u is a positive function in $L^\infty(m)$ and if* $\log u$ *is in $L^1(m)$, then there is a ϕ in H^∞ such that $|\phi|=u$ a.e.*

Now let Y be the maximal ideal space of $L^\infty(m)$, let $\phi\to\hat{\phi}$ be the Gelfand transform from $L^\infty(m)$ onto $C(Y)$, and let $\hat{H}^\infty$ be the image of H^∞. It is a consequence of the preceding theorem that $\hat{H}^\infty$ satisfies the hypotheses of the following lemma concerning function algebras.

LEMMA 1. *Suppose $B\subset C(X)$ is a uniform algebra whose Šilov boundary is X. Suppose U is a nonempty open subset of X and ψ in B does not*

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