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CHEVALLEY GROUPS OVER COMMUTATIVE RINGS[1]

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1. **Introduction.** Steinberg [8] has given a simple presentation for the universal central extension [7], [8], [9] of the group of rational points of a simply connected Chevalley group over a field. In this note we announce a similar theory for the simply connected Chevalley groups over *commutative rings* and outline the proof of a stability theorem for certain functors resulting from this construction. Complete proofs will appear elsewhere.

Let us introduce some notation. A denotes a commutative ring with 1, A^* is its group of invertible elements, $\mathfrak{p}$ and $\mathfrak{q}$ are ideals of A, and $(1+\mathfrak{q})^* = (1+\mathfrak{q})\cap A^*$. Φ is a reduced irreducible root system [2] and $G(\Phi,\ )$ is the simply connected Chevalley-Demazure group scheme with root system Φ. If Φ is of type C_l, $l\geqq 1$ ($C_1=A_1$), we say Φ is *symplectic*, and if Φ is of type A_l, B_l, C_l, or D_l, we say Φ is *classical*. The subgroup of $G(\Phi, A)$ generated by the elementary unipotents $e_\alpha(t)$, $\alpha\in\Phi$, $t\in A$, will be denoted $E(\Phi, A)$. A full discussion of these notions may be found in [3], [5], and [9].

Define the Steinberg group, $\mathrm{St}(\Phi, A)$, to be the group with generators $x_\alpha(t)$, $\alpha\in\Phi$, $t\in A$, subject to the relations

$$(1.1)\quad \begin{aligned} x_\alpha(s)x_\alpha(t) &= x_\alpha(s+t) && (\alpha\in\Phi;\ s,t\in A)\\ [x_\alpha(s), x_\beta(t)] &= \prod x_{i\alpha+j\beta}(N_{\alpha,\beta,i,j}s^it^j) && (\alpha,\beta\in\Phi,\ \alpha+\beta\neq 0)\end{aligned}$$

where the product is as in [8]. Since the elementary unipotents $e_\alpha(t)$ also satisfy these relations, the map $x_\alpha(t)\mapsto e_\alpha(t)$ extends to a homomorphism $\pi:\mathrm{St}(\Phi, A)\to G(\Phi, A)$ with image $E(\Phi, A)$. Set $\ker \pi = L(\Phi, A)$.

In §2 we present certain commutator formulas which yield necessary and sufficient conditions for $E(\Phi, A)$ and $\mathrm{St}(\Phi, A)$ to be their own derived groups. In §3 we show that the extension $\mathrm{St}(\Phi, A)\to E(\Phi, A)$

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