BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 77, Number 2, March 1971

FINITELY GENERATED NILPOTENT GROUPS WITH ISOMORPHIC FINITE QUOTIENTS

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Communicated by Armand Borel, August 10, 1970

Let G be a finitely generated nilpotent group and let $\mathfrak{F}(G)$ denote the set of isomorphism classes of finite homomorphic images of G. If H is another finitely generated nilpotent group, we will say that G and H have isomorphic finite quotients if $\mathfrak{F}(G)=\mathfrak{F}(H)$. The finite quotients of a finitely generated nilpotent group provide much information about the structure of the group [**3**], [**5**], although they do not determine the group up to isomorphism [**8**, and G. Higman, unpublished]. The following result shows, however, that a finitely generated nilpotent group is determined to a large extent by its finite quotients.

THEOREM. *Let G be a finitely generated nilpotent group. Then the finitely generated nilpotent groups H, for which $\mathfrak{F}(G)=\mathfrak{F}(H)$, lie in only finitely many isomorphism classes.*

This theorem, which is a much stronger version of an unpublished result of A. Borel, is proved by using the Lie algebras of the respective nilpotent groups [**6**], [**7**] to apply some finiteness results for arithmetic subgroups of algebraic groups [**4**], a technique introduced by Auslander and Baumslag [**1**], [**2**].

OUTLINE OF THE PROOF. We first give some necessary notation and state a few fundamental lemmas. If G is a finitely generated nilpotent group, we can define a p-adic topology on G for which a neighborhood basis of the identity is given by the groups $G^{p^i}=\mathrm{gp}\{x^{p^i}\mid x\in G\}$. The completion of G in this topology will be denoted Z_pG. The connection between these completions and finite quotients is given by the following lemma of Borel:

LEMMA 1. *If G and H are finitely generated nilpotent groups, then $\mathfrak{F}(G)=\mathfrak{F}(H)$ iff Z_pG and Z_pH are isomorphic for each prime p.*

If N is a subgroup of G, then Z_pN may be considered to be a sub-

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AMS 1969 *subject classifications*. Primary 2040, 2027.

Key words and phrases. Isomorphic finite quotients, arithmetic groups, algebraic groups, Lie algebras.

[1] A generalization of part of the results of the author's doctoral dissertation written under the guidance of G. Baumslag at Rice University, where the author was supported by a Rice Fellowship.