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AN EXISTENCE THEOREM FOR SURFACES OF CONSTANT MEAN CURVATURE

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I. **Introduction.** Let γ be an oriented, rectifiable Jordan curve in E^3 homeomorphic to the unit circle, $u^2+v^2=1$. Let Δ be the open unit disk, $u^2+v^2<1$, and let $\bar{\Delta}$ be its closure. The classical existence theorem for Plateau's problem as proven by J. Douglas [1], and T. Rado [6] asserts the existence of a minimal surface of the type of the unit disk, whose boundary is γ, and which has minimum Lebesgue area. The theorem stated in this paper is an extension of this result to surfaces of constant mean curvature.

Let $h(u, v):\bar{\Delta}\to E^3$ be a given minimal surface solving Plateau's problem. Let K be a given constant and consider the class of continuous vector functions $\mathbf{x}:\bar{\Delta}\to E^3$ whose boundary values describe γ, and such that the oriented volume enclosed by $\mathbf{x}$ and h is K. We prove that in this class there is an $\mathbf{x}$ of minimum Lebesgue area. $\mathbf{x}$ is a representation of a surface of constant mean curvature and satisfies the following system of equations.

$$(1)\qquad \begin{aligned} &\text{(a)}\ \Delta\mathbf{x} = 2H(\mathbf{x}_u \times \mathbf{x}_v),\\ &\text{(b)}\ |\mathbf{x}_u| \equiv |\mathbf{x}_v|,\quad (\mathbf{x}_u\cdot\mathbf{x}_v) = 0 \quad [\text{conformality}],\\ &\text{(c)}\ \mathbf{x}:\partial\Delta \to E^3 \text{ is an admissible representation of } \gamma. \end{aligned}$$

Previous existence theorems for the system (1) have been given by E. Heinz [2], H. Werner [8], and S. Hildebrandt [3]. They proved that if γ is contained in the unit ball, $x^2+y^2+z^2\leqq 1$, and if H with $|H|\leqq 1$ is given, then there exists a solution to the system (1) which is itself contained in the unit ball.

We now give a more precise statement of the theorem.

II. **Statement of theorem.** Denote by $\mathcal{S}(\gamma)$ the set of vector functions $\mathbf{x}:\bar{\Delta}\to E^3$ continuous on $\bar{\Delta}$, continuously differentiable on Δ, whose boundary values are an admissible representation of the oriented Jordan curve γ, and such that the "Dirichlet" integral

$$(2)\qquad D(\mathbf{x}) \equiv \iint_\Delta |\mathbf{x}_u|^2 + |\mathbf{x}_v|^2\, du\, dv$$

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