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ON AN INEQUALITY OF MEAN CURVATURES OF HIGHER DEGREE[1]

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1. **Introduction.** Let $x:M\to E^{n+N}$ be an immersion of an n-dimensional closed manifold M immersed in a euclidean space E^{n+N} of dimension $n+N$. Let B_v be the bundle of unit normal vectors of $x(M)$ so that a point of B_v is a pair (p, e), where e is a unit normal vector to $x(M)$ at $x(p)$. Then B_v is a bundle of $(N-1)$-dimensional spheres over M and is a manifold of dimension $n+N-1$. Let dV be the volume element of M. There is a differential form $d\sigma$ of degree $N-1$ on B_v such that its restriction to a fibre is the volume element of the sphere of unit normal vectors at a point $p\in M$; then $d\sigma\wedge dV$ is the volume element of B_v. For each $(p, e)\in B_v$, there corresponds a symmetric linear transformation $A(p, e)$ of the tangent space $T_p(M)$ of M at p, called the second fundamental form at (p, e). The eigenvalues $k_1(p, e), \cdots, k_n(p, e)$, of the second fundamental form $A(p, e)$ are called the principal curvatures at (p, e). The ith mean curvature $K_i(p, e)$, $i=1, 2, \cdots, n$, are defined by the elementary symmetric functions as follows:

$$(1)\qquad \binom{n}{i} K_i(p, e) = \sum k_1(p, e) \cdots k_i(p, e), \qquad i = 1, 2, \cdots, n,$$

where $\binom{n}{i}=n!/i!(n-i)!$.

We call the integral $K_i^*(p)=\int |K_i(p, e)|^{n/i}\, d\sigma$ over the sphere of unit normal vectors at $x(p)$, the ith total absolute curvature of the immersion x at p, and we define as the *ith total absolute curvature* of M itself the integral $\int_M K_i^*(p)dV$.

In this note, I would like to announce the following results:

THEOREM 1. *Let $x:M\to E^{n+N}$ be an immersion of a closed manifold of dimension n into E^{n+N}. Then we have the following inequality*:

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