BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 77, Number 1, January 1971

EQUATIONAL AXIOMS FOR CLASSES OF LATTICES

BY KIRBY A. BAKER[1]

Communicated by R. S. Pierce, July 27, 1970

1. Equational axiom problems. This note presents a general method for solving a number of problems in the equational theory of lattices. Current interest in this theory stems from Jónsson's important discovery [7], building on earlier work of Birkhoff, that a lattice sharing the algebraic identities common to a given class K of lattices is more tightly bound to K than would be expected for most other kinds of algebraic systems. In this context, a natural question for any such class K is the following "axiom problem."

A.P.(K): *Find a set of equational axioms for* K, *i.e., a set* Σ *of identities, common to the members of* K, *of which all other such identities are lattice-theoretic consequences.*

An equivalent requirement on Σ is that the class of lattices defined by Σ coincide with K^e, the smallest equational class (class definable by identities) containing K.

The problem A.P.(K) is to be viewed as a practical one; the solution is to be constructed explicitly starting from some given definition of K. McKenzie [9], for example, has given just such an explicit solution of A.P.($\{L\}$) for each finite lattice L.

The general method to be developed below solves the axiom problems of all classes of lattices in the following list, among many others.

(a) PP, the class of all projective planes (viewed as lattices of flats). More generally,

(b) $PP(\mathcal{E})$, the class of projective planes subject to a given list $\mathcal{E}$ of excluded configurations. An example is the class of Desarguesian planes, for which solutions to the axiom problem have been given by Schützenberger [10] and Jónsson [6, Theorem 7.1].

(c) $Lth(m)$, the class of all lattices of length at most m, i.e., lattices in which the longest chain has at most $m+1$ elements. This axiom problem was posed by Jónsson, who later solved the case $m=2$ [8].

(d) $Wth(m)$, the class of all lattices of width at most m, i.e., lattices

---

AMS 1970 *subject classifications.* Primary 06A20, 08A15; Secondary 06A30, 08A20.

Key words and phrases. Equational class of lattices, variety of algebras, algebraic identity, lattice polynomial, universal disjunction of equations.

[1] Research supported in part by N.S.F. grants #GP-13164 and #GP-23113.