GROUP DUALITY AND ISOMORPHISMS OF FOURIER AND FOURIER-STIELTJES ALGEBRAS FROM A W^*-ALGEBRA POINT OF VIEW

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0. **Introduction.** In this paper two commutative, semisimple Banach algebras with involution, viz., $A(G)$ and $B(G)$, (called the Fourier algebra of G and the Fourier-Stieltjes algebra of G, respectively) are defined (as in [5]) for an *arbitrary* locally compact topological group G. [*Note.* As algebras $A(G)$ and $B(G)$ are subalgebras of the algebra of bounded continuous functions on G.] The main purpose of this paper is to show that these algebras deeply reflect the structure of G even though they apparently are "trivial" in the sense that multiplication is defined pointwise–not by convolution as, for example, in the case of $L^1(G)$.

The first main result is closely related to a well-known duality theorem, cf. [4], [5], [9], [10], [13], and it has a relatively short proof.

The group G is recovered topologically as a special subset of the spectrum of $B(G)$ (or as the spectrum of $A(G)$), and by virtue of the fact that the dual of $B(G)$ or of $A(G)$, denoted $B(G)'$, $A(G)'$ respectively, can be naturally equipped with an algebraic structure, G can be recovered algebraically as well. The main difference between our Theorem 1 and the above results is that we proceed directly to recovering the group G in the $B(G)$, $B(G)'$ context and deduce the duality theorem in the $A(G)$, $A(G)'$ context as an easy corollary; whereas the other papers attack the $A(G)$, $A(G)'$ duality first and, except for [4] and a long complicated formulation in [13], do not discuss the process of recovering the group from the $B(G)$, $B(G)'$ duality. We also obtain the formula in Theorem 1 (i) and further clarify the relationship between $B(G)$ and the representation theory of G.

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