EQUIVARIANT DYNAMICAL SYSTEMS

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In this note we consider equivariant vector fields and diffeomorphisms and present results which generalise some well-known theorems of the theory of dynamical systems as developed by Smale and others. The main result is a generalisation of the Kupka-Smale density theorem. Proofs will be given elsewhere; the note is a summary of the author's Ph.D. thesis, done at the University of Warwick.

For a survey of dynamical systems theory, see [1]; for elementary facts about equivariant vector fields, see [2].

M will always denote a compact C^∞ manifold, without boundary, and G a compact Lie group acting differentiably on M. Let $GVB(M)$ and $GFB(M)$ respectively denote the categories of C^∞ G-vector bundles and C^∞ G-fiber bundles over M; we assume paracompact fiber.

Thus $TM \in GVB(M)$ in a natural way. For $E \in GVB(M)$ or $GFB(M)$ we may consider $C^r_G(E) = \{X \in C^r(E) : gXg^{-1}(x) = X(x), g \in G, x \in M\}$: The space of equivariant sections of E. It is well known that for $E \in GVB(M)$, $C^r_G(E)$ is a Banach splitting subspace of $C^r(E)$, with respect to the C^r topology on $C^r(E)$.

As a straightforward generalisation of the proof given for $G = \text{id}$ in [3], we have:

THEOREM 1. *If $E \in GFB(M)$, then $C^r_G(E) \subset C^r(E)$, as a closed C^∞ Banach submanifold, in a natural unique way.*

COROLLARY 1.1. $\text{Diff}^r_G(M) \subset C^r(M, M)$, $r \geqq 1$, *as a C^∞ submanifold. Here* $\text{Diff}^r_G(M)$ *denotes the set of C^r equivariant diffeomorphisms of M, with the C^r topology.*

If $X \in C^r_G(TM)$ and $X(x) = 0_x$, then $X(gx) = 0_{gx}$, $g \in G$, and $G(x)$ is a singular set for X. Similarly for $f \in \text{Diff}^r_G(M)$, with $fx = x$.

Let q be a closed orbit of $X \in C^r_G(TM)$. We define G_q, the "isotropy group of q", by:

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