THE ORDER OF THE IMAGE OF THE J-HOMOMORPHISM

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ABSTRACT. This note announces a proof of the order of the image of the J-homomorphism and gives several other results in homotopy theory which are consequences of the proof.

The set $\Omega^n S^n$ can be identified with the set of all base point preserving maps of S^n into itself. $SO(n)$, acting on S^n as R^n with a point at infinity, is also a set of base point preserving maps of S^n onto itself. This defines $SO(n) \subset \Omega^n S^n$. The induced map in homotopy is called the J-homomorphism. If we allow n to go to infinity we have the stable J-homomorphism. By Bott's results [3] $\pi_j(SO) = Z$, $j \equiv -1 \bmod 4$, and $= Z_2 j \equiv 0, 1 \bmod 8$, $j > 0$, and zero otherwise. Adams [1] showed that the Z_2 summand maps monomorphically and Milnor and Kervaire [6] showed that the Z group in dimension $4j-1$ maps nontrivially and its image generates a subgroup of at least a certain order λ'_j. Adams [1] showed that the order was either λ'_j or $2\lambda'_j$ and if $j \equiv 1$ (2) it was λ'_j. Thus only the two primary part is in question and there only for $j \equiv 0$ (2). Let λ_j be the two primary part of λ'_j. If $4j \equiv 2^{\rho(j)} \bmod 2^{\rho(j)+1}$ (which defines $\rho(j)$) then $\lambda_j = 2^{\rho(j)+1}$. We prove:

THEOREM 1. *The 2-primary order of the image of J in stem $4j-1$ is λ_j.*

The proof has several corollary results which have some interest. The first result is rather technical but still has some interest. The naming of elements in $H^{**}(A)$ is that given in [5].

THEOREM 2. *The elements $P^i c_0$, $P^i h_1 c_0$, $i \geqq 1$, $P^i h_2$, $i \geqq 1$, in $H^{**}(A)$ represent the image of J in dimension $j \equiv 0, 1, 3 \bmod 8$. In dimension $8j-1$ the "tower" which ends at the "Adams edge" represents the image of J in that dimension.*

These elements were known to have the desired e-invariant property [1] and were believed to be in J. Their Whitehead product behavior has been investigated ([2] and [4], for example).

Let $M = Z_2 + Z_2$ (be the module over A with one generator; μ in

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