SMOOTHINGS[1] AND HOMEOMORPHISMS FOR HILBERT MANIFOLDS

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0. The aim of this paper is to announce several results concerning smoothing Hilbert manifolds and some of their consequences for homeomorphisms. All manifolds are assumed to be Hausdorff, paracompact, and separable with local model the ∞-dimensional separable Hilbert space H. (All separable infinite dimensional Fréchet spaces are homeomorphic to H.) As usual, we denote by D^∞ the unit disc and by S^∞ the unit sphere in H. A pair $(M, \partial M)$ is called a *differentiable* manifold with boundary if it has a C^∞-differentiable structure with local model the Hilbert half space $H\times[0, \infty)$. The boundary ∂M, is the set of those points which are mapped onto $H\times\{0\}$ by some chart. It can be shown that ∂M is a differentiable manifold and is locally collared and thus collared in M. (See for instance, the existence of closed tubular neighborhoods [11].) This definition is not adequate for *topological* manifolds with boundary, because by Klee [10], $H\times[0, \infty)$ is homeomorphic to H. We thus define a topological manifold with boundary to be a pair $(M, \partial M)$ such that (a) M and ∂M are topological manifolds, (b) ∂M is collared in M, or equivalently locally collared (according to M. Brown, *Topology of 3-manifolds and related topics*, edited by M. K. Fort, p. 88). Our results are the following:

THEOREM 0.1. *Any homotopy equivalence of pairs $f:(N, \partial N)\to(M, \partial M)$ between differentiable (topological) manifolds with boundary is homotopic to a diffeomorphism (homeomorphism) h of pairs; moreover, $h/\partial N$ can be any diffeomorphism $l:\partial N\to\partial M$ homotopic in ∂M to $f/\partial N$ or any homeomorphism $l:\partial N\to\partial M$ homotopic in M to $f/\partial N$.*

In the differentiable case this theorem is a consequence of [3] or [7]. In the topological case, since M and N are topological manifolds with the same homotopy type, there exists a homeomor-

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