SELF-INTERSECTIONS IN CONTINUOUS RANDOM WALK

BY Z. A. MELZAK[1]

Communicated by H. P. McKean, Jr., June 29, 1970

An n-step random walk ($n \geqq 3$) is a sequence of n straight segments, called steps, in the plane; each step is of length 1, the first step starts at the origin and each successive step starts at the end of the previous one; every step is in random direction with uniform distribution in angle. Neglecting certain events of probability 0 we define a self-intersection as the event when for some i and j, with $1 \leqq i < j \leqq n$ and $j-i>1$, the ith and the jth step have in common exactly one point, interior to each step. Let $f(n)$ be the expected number of self-intersections; it is proved that

$$f(n) = \frac{n}{4}\sum_{p=2}^{n-1}\left(1-\frac{p}{n}\right)$$

$$\Bigg[1 - \frac{4}{\pi^2}\int_0^\infty\int_0^\infty (uv)^{-1}J_0(v)\cdot[J_0^{p-1}(u-v) - J_0^{p-1}(u+v)]du\,dv$$

$$(1) \qquad + \frac{1}{\pi^5}\int_0^\infty\int_0^\infty\int_0^\infty\int_0^\infty\int_0^{2\pi}(uvwz)^{-1}\sum_{i=1}^{8}\epsilon_i\cos(c_i\sin\theta)$$

$$\cdot J_0^{p-1}((a_i^2 + 2\eta_i a_i b_i\cos\theta + b_i^2)^{1/2})d\theta\,du\,dv\,dw\,dz\Bigg],$$

where J_0 is the Bessel function of the first kind and zero order, and the quantities indexed by i are as given below:

(2)

i	ϵ_i	η_i	a_i	b_i	c_i
1	1	-1	$w-z$	$u-v$	$v-w$
2	1	1	$w-z$	$u-v$	$v+w$
3	-1	-1	$w-z$	$u+v$	$v+w$
4	-1	1	$w-z$	$u+v$	$v-w$
5	-1	-1	$w+z$	$u-v$	$v-w$
6	-1	1	$w+z$	$u-v$	$v+w$
7	1	-1	$w+z$	$u+v$	$v+w$
8	1	1	$w+z$	$u+v$	$v-w$.

AMS 1970 *subject classifications*. Primary 6066.

Key words and phrases. Random walk, self-crossings.

[1] Research reported upon was supported by the N. R. C. of Canada.