THE NONFINITE TYPE OF SOME $\mathrm{Diff}_0 M^n$

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Communicated by Louis Auslander, May 1, 1970

1. **Introduction.** Throughout this paper, unless explicitly stated otherwise, all manifolds will be closed, connected, oriented, and of class C^∞. We denote by X some arbitrary closed subset of the n-manifold M^n, $X \neq M^n$. When X is empty, we shall suppress it from the notation. The set of orientation-preserving self-diffeomorphisms of M^n that fix each point of X can be made into a locally-path-connected, metrizable, topological group $\mathrm{Diff}(M^n, X)$ by endowing it with the usual C^∞ topology and the operation of map-composition [6], [8]. Let $\mathrm{Diff}_0(M^n, X)$ be the identity component of $\mathrm{Diff}(M^n, X)$. For general M^n, the only global homotopy-theoretic fact known about $\mathrm{Diff}_0(M^n, X)$ is that it has the homotopy type of a countable CW complex [8].

Let S^n be the standard, oriented n-sphere. In [1], the authors announced that $\mathrm{Diff}_0 S^n$ does not have the homotopy type of a finite CW complex when $n \geqq 7$. The techniques described in §3 of this announcement, together with [1], allow us to extend this result to other manifolds.

2. **Statement of the main results and remarks.** Our main result is the following:

2.1. THEOREM. *If M^n is a spin manifold with trivial rational Pontrjagin classes, then $\mathrm{Diff}_0(M^n, X)$ does not have the homotopy type of a finite* CW *complex when either* (a) $n=8k-4$, $k \geqq 6$, *or* (b) $n=8k$ *and k is admissible.*

An *admissible natural number k* is a natural number $\geqq 42$ for which the open interval $(\frac{1}{3}(2k+1), \frac{1}{2}(2k+1))$ contains at least one prime. It follows from the Prime Number Theorem that there are at most finitely many *in*admissible natural numbers, but the precise value of the largest such number is not known. See Remark 2.5 below.

2.2. REMARK. It is clear that π-manifolds of the appropriate dimensions satisfy the hypotheses of 2.1. Note that this includes homotopy spheres, homotopy tori, real Stiefel manifolds, compact Lie

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AMS 1969 *subject classifications.* Primary 5720, 5755; Secondary 5372.

Key words and phrases. Homotopy type of CW-complex, π-manifold, space of diffeomorphisms of a manifold, flat manifold, manifolds of constant negative curvature, rational Pontrjagin classes, spin structure.

[1] The first two authors were supported in part by National Science Foundation grant GP-7952X1 and the third was supported by a National Science Foundation Postdoctoral Fellowship.