

RESEARCH ANNOUNCEMENTS

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STRUCTURE OF INCIDENCE ALGEBRAS AND THEIR AUTOMORPHISM GROUPS¹

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Let P be a locally finite ordered set, i.e., a (partially) ordered set for which every segment $[X, Y] = \{Z \mid X \leq Z \leq Y\}$ is finite. The *incidence algebra* $I(P)$ of P over a field K is defined [2] as the algebra of all functions from segments of P into K under the multiplication (convolution)

$$fg(X, Y) = \sum_{Z \in [X, Y]} f(X, Z)g(Z, Y).$$

(We write $f(X, Y)$ for $f([X, Y])$.) Note that the algebra $I(P)$ has an identity element δ given by

$$\begin{aligned}\delta(X, Y) &= 1, & \text{if } X = Y, \\ &= 0, & \text{if } X \neq Y.\end{aligned}$$

THEOREM 1. *Let P and Q be locally finite ordered sets. If $I(P)$ and $I(Q)$ are isomorphic as K -algebras, then P and Q are isomorphic.*

SKETCH OF PROOF. The idea is to show that the ordered set P can be uniquely recovered from $I(P)$. Let the elements of P be denoted X_α , where α ranges over some index set. Then a maximal set of primitive orthogonal idempotents for $I(P)$ consists of the functions e_α defined by

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