

HIGHER DERIVATIONS AND AUTOMORPHISMS OF COMPLETE LOCAL RINGS

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ABSTRACT. This paper begins with a review of those aspects of the theory of higher derivations on fields which form a background for the study of recent uses of higher derivations in automorphism theory of complete local rings. Basic definitions and basic properties of convergent higher derivations on complete local rings are discussed including the concept of convergent rate group of automorphisms, the theory of which is at the present time almost totally undeveloped.

Methods of constructing automorphisms using higher derivations are considered next, particularly in connection with the problem of identifying the factor groups of the higher ramification series of a complete local ring. Recent results on this problem are discussed as well as some possible directions for future research on the topics of this article.

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I. Introduction. My purpose in this article is to provide a selective outline of the development of the theory of higher derivations leading to applications in the automorphism theory of complete local rings. As a result certain recent developments in higher derivation theory, except perhaps for casual reference, are outside the scope of this paper, e.g., applications to Galois theory of fields [5], [6], [21], [35], [38] as well as the theory of universal higher derivations and in-

An invited address delivered to the 669th meeting of the Society at Baton Rouge, Louisiana on November 22, 1969; received by the editors May 25, 1970.

AMS 1969 subject classifications. Primary 1360, 1395; Secondary 1393, 1660.

Key words and phrases. Derivation, higher derivation, derivation automorphism, inertial automorphism, convergent rate group, v -ring, complete regular local ring, ramification, ramification series, p -basis.