POLYTOPES, GRAPHS, AND COMPLEXES[1,2]

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During the last few years the theory of convex polytopes has been developing at an ever accelerating pace. As a consequence, many parts of the book "Convex Polytopes" ([CP] = Grünbaum [**1967a**]), which was essentially completed early in 1966, are by now completely out of date. The same fate befell the surveys by Klee [**1966**] and Grünbaum [**1967b**]; even the more recent survey by Grünbaum-Shephard [**1969**] has been overtaken in many directions by new results. The new achievements are scattered in the literature, and many of them are still in the preprint stage. Hence it seemed worthwhile to prepare an up-to-date summary of the new results, organized in a form that makes it readable without previous specialized knowledge of the theory of polytopes.

The following pages constitute such a survey of a part of the theory; considerations of time and space precluded coverage of all ramifications of the subject of convex polytopes, but most major directions are included. The first part of the survey deals with problems and questions related to graphs, and underscores the extent to which the investigations of polytopes and graphs influence each other. The second part deals with relations among the numbers of

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