FULLY NUCLEAR OPERATORS

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1. Introduction. This note is an outgrowth of a study of the following conjecture of Grothendieck [5, Chapter II, p. 47].

(C) Let E and F be Banach spaces such that every $T \in \mathcal{L}(E, F)$, the continuous linear operators from E to F, is nuclear. Then either E or F is finite dimensional.

Recall that an operator $T: E \to F$ is *nuclear* if there exists $(f_i) \subset E'$, $(x_i) \subset F$ such that

$$Tx = \sum_{i=1}^{\infty} f_i(x) x_i \qquad \text{for every } x \in E \quad \text{and} \quad \sum_{i=1}^{\infty} \|f_i\| \, \|x_i\| < +\infty.$$

Every nuclear operator obviously has the property that the image of an unconditionally convergent series is absolutely convergent. An operator with this latter property is called an *absolutely summing* operator [14] (Grothendieck called these operators "semi-intégrale à droit"). It can happen in nontrivial cases, e.g. $\mathcal{L}(l_1, l_2)$, that all continuous linear operators are absolutely summing [5], [9]. Thus, one approach to (C) is to seek a criterion guaranteeing the existence of nonabsolutely summing operators between Banach spaces. Such a criterion is developed in [22].

To our knowledge (C) has been considered only in [2] and the important recent paper of Lindenstrauss and Pełczyński [9]. Although we are unable to solve (C) in the generality asserted by Grothendieck, we are able to give an affirmative answer to (C) under a somewhat more restricted hypothesis. However, we should mention that if (C) is true as asserted by Grothendieck then our hypotheses must be satisfied.

We need two results from [22].

THEOREM A. *Let E and F be infinite dimensional Banach spaces. Then there is an infinite dimensional subspace E_0 of E and an operator $T: E_0 \to F$ such that T is not absolutely summing.*

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