TRIANGULATED INFINITE-DIMENSIONAL MANIFOLDS

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In this paper we extend almost all the results on infinite-dimensional Fréchet manifolds to apply to manifolds modeled on some l_2^f ($=\{x\in l_2 \mid$ at most finitely many of the coordinates of x are nonzero$\}$) and we show (Theorem 14) that each l_2^f-manifold has a unique completion to an l_2-manifold. (We use l_2 to stand for the Hilbert space of all square-summable sequences of some infinite cardinality $\mathfrak{A}$. When we wish to be more specific, we write $l_2(\mathfrak{A})$.) Examples of $l_2^f(\aleph_0)$-manifolds include the metric S^∞ (the unit sphere in $l_2^f(\aleph_0)$) and the metric RP^∞, which may be regarded as the orbit space of S^∞ acted upon by the antipodal map. (The identity map from S^∞ or RP^∞ with the weak topology to the metric topology is a homotopy equivalence. (see [2]).) See also Theorem 16. In fact, we show (Theorems 15 and 17) that each l_2^f-manifold is a metric (see [2] for definition) simplicial complex and that each $l_2^f(\aleph_0)$-manifold is the 'metric' direct limit of finite-dimensional, closed, orientable manifolds. We conjecture that *the metric geometric realization of each connected singular s.s. complex (or Kan s.s. complex) is an $l_2^f(\mathfrak{A})$-manifold, for some cardinal* $\mathfrak{A}$. Most of the results here have been independently proved for the case of $l_2^f(\aleph_0)$ by T. A. Chapman [1] who used different methods. All manifolds in this paper are assumed paracompact.

DEFINITIONS. (1) If F is a TVS, define F^ω to be the countably infinite product and $F_f^\omega=\{\{x_i\}\in F^\omega \mid$ for at most finitely many i, $x_i\neq 0\}$.

(2) Let X and Y be spaces, $\mathfrak{U}$ be an open cover of Y, and $\mathfrak{F}$ a set of functions from X into Y. Then $\mathfrak{F}$ is said to be *$\mathfrak{U}$-small* if for each $x\in X$ there is a $U\in\mathfrak{U}$ containing $\{f(x) \mid f\in\mathfrak{F}\}$. Members of $\mathfrak{F}$ are said to be *$\mathfrak{U}$-approximate*. A homotopy $F:X\times I\to Y$ is said to be *$\mathfrak{U}$-small* if $\mathfrak{F}=\{F(\ ,t) \mid t\in I\}$ is. A function $g:X\to Y$ is said to be *approximated*

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