AN APPLICATION OF COMPLEX BORDISM TO THE STABLE HOMOTOPY GROUPS OF SPHERES

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In this note we announce several results that are a continuation of the study initiated in [1] of the internal properties of the complex bordism homology functor applied to finite complexes and their external applications. The present work is an outgrowth of our attempts to better understand the results of [1] and [2], especially [1, §6] and [2, §§2–3] that deal with the study of

$$\operatorname{Im}\{\Omega_*^{fr}(X) \to \Omega_*^U(X)\},$$

X a finite complex. This has brought us into contact with many new and interesting questions concerning the realizability of certain cyclic Ω_*^U-modules as complex bordism modules of finite complexes. Needless to say it has also involved us in the stable homotopy groups of spheres, particularly with the occurrence of certain types of infinite families of elements in the p-component (see the discussion below).

The particular Ω_*^U-modules that are of interest to us will require some preparation to describe. Let us therefore fix an odd prime integer p. We recall that there are U-manifolds V^{2p^i-2}, of dimension $2p^i-2$, all of whose mod p Chern numbers vanish, but are acceptable polynomial generators for Ω_*^U in dimension $2p^i-2$ [3], [4]. What we seek is for each nonnegative integer n a finite complex $V(n)$ such that

$$\tilde{\Omega}_*^U(V(n)) \cong \Omega_*^U/(p, [V^{2p-2}], \cdots, [V^{2p^n-2}])$$

as an Ω_*^U-module.

For $n=0$ we may choose for the space $V(0)$ the Moore space $S^1\cup_p e^2$ where $p: S^1 \to S^1$ is a map of degree p. We then know (see e.g. [2, §3]) that

$$\operatorname{Im}\{\tilde{\Omega}_*^{fr}(V(0)) \to \tilde{\Omega}_*^U(V(0))\}$$

consists of the subgroup generated by the elements

$$\{[V^{2p-2}]^t\sigma \mid t = 0, 1, \cdots\}$$

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