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LORENTZIAN SYMMETRIC SPACES¹

BY M. CAHEN AND N. WALLACH

Communicated by Armand Borel, August 25, 1969

0. Introduction. M. Berger has classified the pseudo-riemannian symmetric spaces which are isotropy irreducible [1]. The notion of isotropy irreducibility is too restrictive and the proper building blocks are the indecomposable symmetric spaces (cf. §1). In other words, every pseudo-riemannian symmetric space is locally a product of indecomposable ones. In this paper we announce a complete classification of Lorentzian symmetric spaces; we emphasize the solvable case since the solvable and semisimple case separate (see Theorem 3). It has been conjectured that there were no pseudo-riemannian symmetric spaces with nilpotent isometry group: a class of examples is presented in §4. §4 also contains examples of Lorentzian symmetric tori, analogous to flat tori.

Our interest in this problem originated with the study of Huyghens' principle in general relativity [2]; the 4 dimensional solvable Lorentzian symmetric spaces satisfy Huyghens' principle.

We would like to thank J. Tits for the term indecomposable, C. C. Moore for pointing out that the Heisenberg algebra was related to our algebra \mathfrak{g}_A and A. Taub and G. Walker for stimulating discussions.

1. Pseudo-riemannian symmetric quadruples. By a pseudo-riemannian symmetric space we mean a connected, C^∞ manifold, $(M, \langle \ , \ \rangle)$

AMS Subject Classifications. Primary 5373, 5378.

Key Words and Phrases. Pseudoriemannian symmetric spaces, indecomposable, symmetric quadruple, semisimple Lie algebra, solvable Lie algebra, Riemann tensor.

¹ This work was supported in part by Air Force Office of Scientific Research, Office of Aerospace Research, United States Air Force, under AFOSR Grant No. 903-67 and National Aeronautics and Space Administration under NASA Grant No. NGL 44-004-001.