PERTURBING ALMOST PERIODIC DIFFERENTIAL EQUATIONS[1]

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ABSTRACT. When the zero solution of an almost periodic differential system is uniform-asymptotically stable, it is shown that the zero solution is diminishingly asymptotically stable and totally asymptotically stable. Generalization of the above theorem to bounded solutions and functional differential equations with finite time-lag are obtained by the same method. The techniques used in the proof also give a more general approach to perturbation theory of differential equations.

1. Consider a system of differential equations

$$x' = f(t, x) \tag{E}$$

and its perturbed equations

$$x' = f(t, x) + p(t). \tag{P}$$

We always assume at least that:

(i) $f(t, x)$ is defined on $[0, \infty)\times S_c$, where $S_c=\{x\in R^d: |x| <c, c>0\}$, such that $f(\cdot, x)$ is measurable for each x, $f(t, \cdot)$ is continuous for each t, and f is bounded on every compact subset of $[0, \infty)\times S_c$;

(ii) $p(t)$ is defined and measurable on $[0, \infty)$.

In a recent paper Strauss and Yorke [2] have shown (for definitions, see §4)

THEOREM 1. *Suppose the origin* 0 *is uniform-asymptotically stable* (UAS) *for* (E), *and f belongs to the class of Lipschitz or linear or inner-product or periodic functions. Then for every absolutely diminishing $p(t)$, the origin* 0 *is eventually uniform-asymptotically stable* (EvUAS) *for* (P).

For a detailed discussion of Theorem 1, refer to [2].

A natural question is: does this theorem hold if $f(t, x)$ is almost periodic? An answer is given in Theorem 2.

2. Define $f_\tau(t, x)=f(t+\tau, x)$ and $H(f)=\{f_\tau: \tau\in R\}$. $\overline{H}(f)$ denotes the closure of $H(f)$ in the sense of uniform convergence on all of $R\times S$, for any compact subset $S\subset S_c$.

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