BOUNDING IMMERSIONS OF CODIMENSION 1 IN THE EUCLIDEAN SPACE

BY ANDRÉ GRAMAIN

Communicated by Emery Thomas, August 4, 1969

Let M be an $(n+1)$-dimensional differentiable manifold without boundary (compact or not) and $f\colon V\to M$ an immersion of the compact n-dimensional manifold without boundary V. We say that f is a *bounding immersion* if there is a manifold W^{n+1} with boundary $dW=V$, and an immersion $g\colon W\to M$ such that $f=g|V$. If M and V are oriented, then V must be the oriented boundary of the oriented manifold W, and g an oriented immersion of codimension 0.

Using the classification of immersions (Smale [7], Hirsch [2]) and the work of Kervaire-Milnor [3], [4], we compute in this note the regular homotopy classes of all bounding immersions of the sphere S^n into the euclidean space R^{n+1} and into the sphere S^{n+1}.

1. Statement of the results. From [2] we know that the derivation $f\mapsto T(f)$ defines a weak homotopy equivalence between the space $\mathrm{Imm}(V, M)$ of the immersions of V into M and the space of the fibre-maps of the tangent bundle $T(V)$ into the tangent bundle $T(M)$ which are injective in each fibre. If $V=S^n$ and $M=R^{n+1}$, the set of connected components of this last space is an homogeneous space under the group $\pi_n(\mathrm{SO}(n+1))$. By a convenient identification, we obtain a bijection $\gamma\colon\pi_0(\mathrm{Imm}(S^n, R^{n+1}))\to\pi_n(\mathrm{SO}(n+1))$ such that the class of the ordinary imbedding be $0\in\pi_n(\mathrm{SO}(n+1))$. Furthermore the map γ is additive with respect to the connected sum of immersions [5].

Similarly, using the fact that the fibration $\mathrm{SO}(n+2)\to S^{n+1}=\mathrm{SO}(n+2)/\mathrm{SO}(n+1)$ is the principal fibration with group $\mathrm{SO}(n+1)$ tangent to S^{n+1}, it is easy to obtain a bijection $\beta\colon\pi_0(\mathrm{Imm}(S^n, S^{n+1}))\to\pi_n(\mathrm{SO}(n+2))$ additive with respect to the connected sum. If $i\colon R^{n+1}\to S^{n+1}$ is the stereographic projection with the south pole $(x_1=-1)$ as center, we have a commutative diagram

$$\begin{array}{ccc}\pi_0(\mathrm{Imm}(S^n, R^{n+1})) & \xrightarrow{\gamma} & \pi_n(\mathrm{SO}(n+1))\\ \downarrow i_* & & \downarrow s\\ \pi_0(\mathrm{Imm}(S^n, S^{n+1})) & \xrightarrow{\beta} & \pi_n(\mathrm{SO}(n+2))\end{array}$$

AMS Subject Classifications. Primary 5720.

Key Words and Phrases. Differentiable immersion, bounding immersion, codimension one, Smale classification, Hopf-Whitehead homomorphism, Kervaire-Milnor exact sequence.