A CONJECTURE CONCERNING TRANSITIVE SUB-ALGEBRAS OF LIE ALGEBRAS

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This is to announce the settling of the following conjecture: Given a Lie pseudogroup [1] acting transitely on a manifold, is there a finite-dimensional subgroup which also acts transitively? The answer is, in general, no. We give here an example and, in addition, give the Jordan-Hölder decomposition of a large class of counter-examples. Finally, we show how these counterexamples occur among general transitive pseudogroups. Following [1] and [2], we work in the category of transitive (filtered) Lie algebras. Details will appear in a forthcoming paper [3].

A transitive algebra L is called *minimal* if, given a transitive subspace T [1], L is the smallest transitive subalgebra generated by T.

THEOREM 1. *Every minimal ideal* [2] *of a minimal transitive Lie algebra is abelian.*

According to the results of [2], this theorem is proved if it can be shown that a minimal ideal cannot be (a) a simple transitive Lie algebra or (b) a simple intransitive Lie algebra. This is accomplished for (a) by using the results of [4] and for (b) by applying the spectral sequence for ideals in Lie algebras [5] together with some of the techniques of [4]. The classification of the simple infinite-dimensional Lie algebras [6] is used repeatedly.

Using Theorem 1 it is not hard to prove

THEOREM 2. *Every minimal transitive Lie algebra L has the following Jordan-Hölder decomposition*:

$$L \supset I_1 \supset I_2 \supset I_3 \supset \cdots I_s \supset I_{s+1} = \{0\},$$

where I_n/I_{n+1} is abelian and L/I_1 is either a simple Lie algebra or a finite-dimensional abelian Lie algebra.

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