# GLOBAL THEOREMS FOR CLOSED PLANE CURVES 

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Let $X$ be a closed plane curve given by a four times continuously differentiable map $X: C \rightarrow R^{2}$ from the circle $C$ into Euclidean twodimensional space $R^{2}$. The results announced here are typified by

Theorem 1. Under certain regularity conditions, the number of straight lines which are tangent to $X$ at two points s and $t$ of $X$ and such that the unit normals to $X$ at s and $t$ are equal is equal to the number of straight lines which are tangent to $X$ at two points s and $t$ of $X$ and such that the unit normals to $X$ at s and t are unequal, plus the number of selfintersections of $X$, plus one-half the number of inflection points of $X$. In Figure 1 the double tangents of the first mentioned kind are drawn solid and the others are dashed.


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