NOTES TOWARDS THE CONSTRUCTION OF NONLINEAR RELATIVISTIC QUANTUM FIELDS. III: PROPERTIES OF THE C^*-DYNAMICS FOR A CERTAIN CLASS OF INTERACTIONS

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1. This note treats the C^*-dynamics of positive-energy symmetric (or 'Bose-Einstein') quantum fields in continuation of [1]. The temporal development of the systems being considered is given by a one-parameter group of automorphisms of a C^*-algebra, which in general are not unitarily implemented, but may by a process of localization be reduced to the consideration of a complex of putative one-parameter unitary groups. Each such group is to be generated by an operator H' which is formally given as $H+V$, where each of H and V may be formulated as a selfadjoint operator in Hilbert space, but whose sum is *a priori* ill-defined as such because of the singular nature of V in relation to H.

In [1, I] a theory of renormalized products of quantum fields was initiated which served as a basis for the treatment of the operators V of concrete interest. It followed that for a certain class of relativistic cases:

(a) $H+V$ is densely defined and has a selfadjoint extension H';

(b) the associated complex of one-parameter unitary groups corresponds to a C^*-automorphism group provided the Lie formula: $e^{itH'}=\lim_n(e^{itH/n}e^{itV/n})^n$ is applicable (as is the case e.g. if H' is unique, by a theorem of Trotter). In the present note, by making a natural use of mild particularities of the operators in question, a selfadjoint extension H' is constructed which has the modified property, sufficient for the construction of an appropriate C^*-automorphism group, that $e^{itH'}=\lim_m\lim_n(e^{itH/n}e^{itf_m(V)/n})^n$, if $\{f_m\}$ is any sequence of real functions of compact support on R^1 such that $f_m(\lambda)\to\lambda$ and $|f_m(\lambda)|\leqq|\lambda|$; and this operator has in addition many other relevant properties. The treatment is quite general, and apart from the finiteness of the moments of V and e^{-V}, and the nonvanishing of the 'mass,' makes no significant assumptions.

2. While the chief goal of the present notes is the construction of nonlinear relativistic quantum fields, it is of mathematical as well as potentially of physical interest to explore other possible formulations of space-time. In partially colloquial terms, the situation may

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