SECOND ORDER NONLINEAR OSCILLATIONS

BY C. V. COFFMAN[1] AND J. S. W. WONG

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We are concerned here with the oscillatory behavior of solutions of the following second order nonlinear ordinary differential equation

$$y'' + yF(y^2, x) = 0, \qquad x > 0, \tag{1}$$

where $yF(y^2, x)$ is continuous for $x>0$ and $|y| < \infty$, and $F(t, x)$ is nonnegative for $x>0$ and $t \geqq 0$. The prototype of equation (1) is the so-called generalized Emden-Fowler equation:

$$y'' + q(x)\,|\,y\,|^\gamma \operatorname{sgn} y = 0, \qquad x > 0, \tag{2}$$

where $q(x) \geqq 0$ and $\gamma > 0$. Here we are interested in the determination of sufficient conditions for the existence of at least one oscillatory solution, and also in the complementary problem, i.e., the determination of conditions which guarantee the nonexistence of oscillatory solutions of (1).

The problem of oscillation and nonoscillation of solutions of equations (1) and (2) has been of great interest in recent years; we refer the reader to [7] for a current survey of the literature on this subject. In most of these studies, it is convenient to subdivide the discussion according to the nonlinear character of equation (2). We say that equation (2) is in the sublinear case if $\gamma \leqq 1$ and it is in the superlinear case if $\gamma \geqq 1$. More generally, one says that equation (1) is in the sublinear case if $F(t, x)$ is monotone decreasing in t and it is in the superlinear case if $F(t, x)$ is monotone nondecreasing in t. Results in the same direction as ours may be found in Belohorec [1] for equation (2) when $\gamma < 1$, and in Jasny [3], Kurzweil [5], Kiguradze [4], and Nehari [6] for equation (2) when $\gamma > 1$. For the more general equation (1), the superlinear case has been considered in Nehari [6], and in Coffman and Wong [2], but as far as we know the corresponding sublinear case has not been investigated. In the present work, we attempt to present a unified treatment for the study of this specific oscillation problem both in the sublinear and the superlinear case. The main results include oscillation and nonoscillation theorems for both of the two classes of equation (1) and contain as special cases all of the results cited above. In fact, in the process of this generalization

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